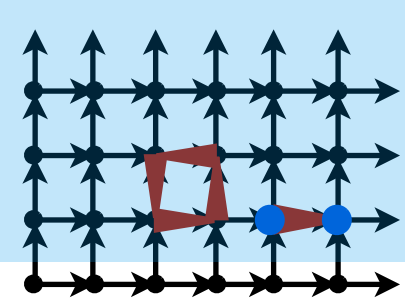


Lattice QCD: Past, present, & future

Stephen R. Sharpe
University of Washington



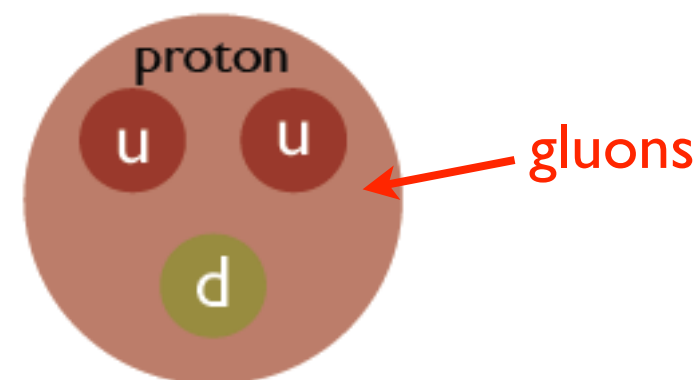
Workshop on Lattice QCD, SNU, 5/23/25



Overview

- Quantum ChromoDynamics is a peculiar theory

- Quarks are absolutely confined
- Quark properties are obscured



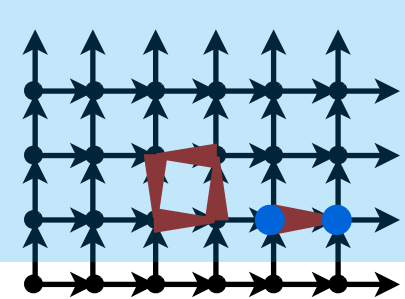
- QCD is a “background” in search for new physics

- Must understand “old physics” of the standard model (SM) to find “new physics”

- QCD is strongly coupled, non-perturbative

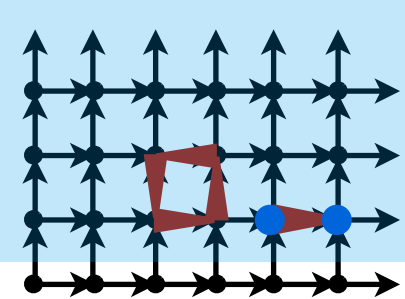
- Lattice QCD allows us to make precise predictions for a range of properties of QCD

- Recent extensions of LQCD methods greatly expand range of predictions



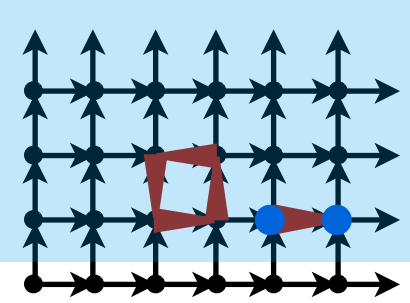
Workshop overview

- This talk: “Lattice QCD: Past, present, & future”
 - Introduction & overview
- Prof. Aida El-Khadra: “Finding Beauty: The role of lattice QCD in precision physics”
 - Applications of lattice QCD (LQCD) to charm and b physics
- Prof. Maarten Golterman: “Frontiers of Lattice Gauge Theory: muon $g-2$ and lattice chiral gauge theories”

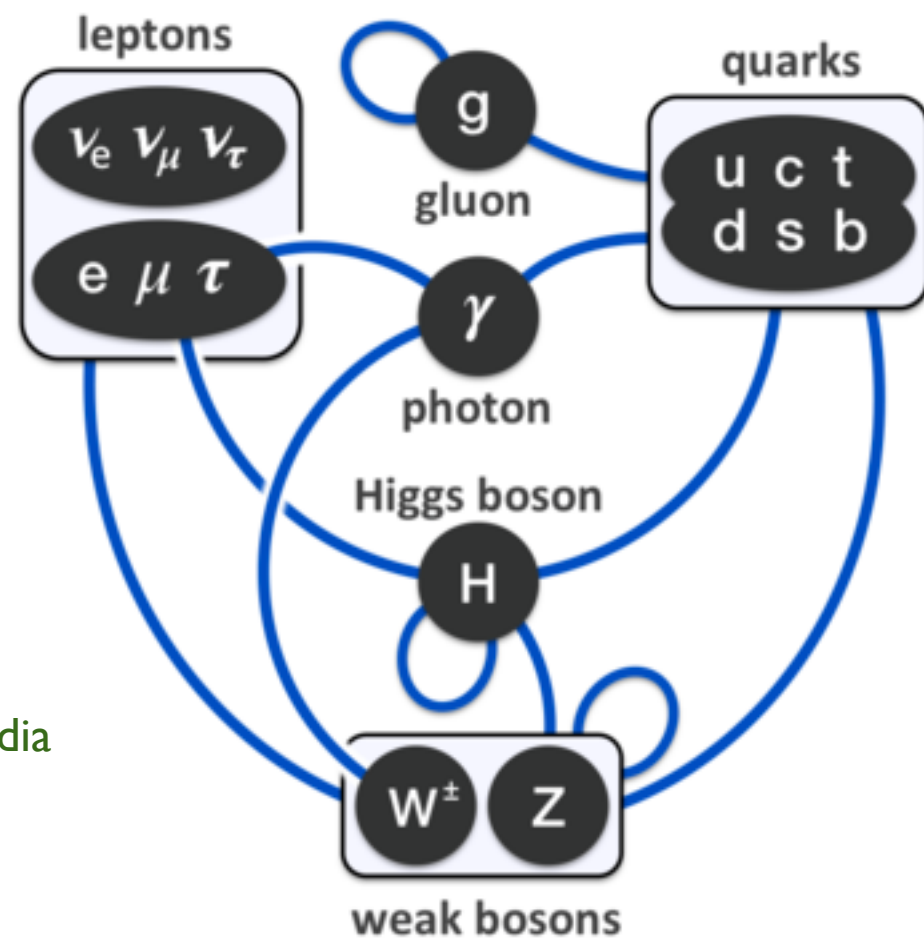


Outline of this talk

- Standard model & searching for physics beyond
- QCD & Lattice QCD
- High precision LQCD
- Constraining the Standard Model with LQCD
- Extending the LQCD frontier
- Summary

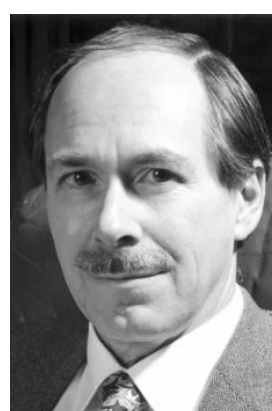


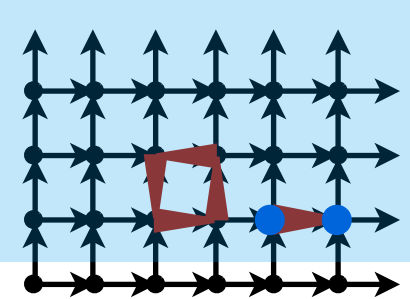
Standard Model (SM)



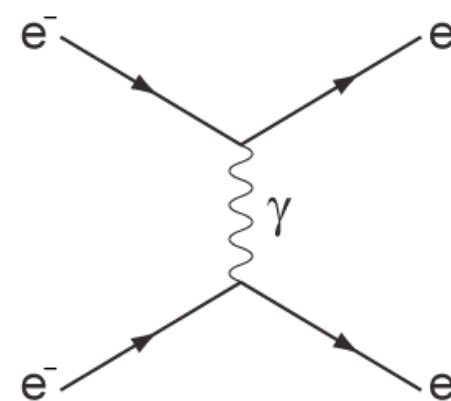
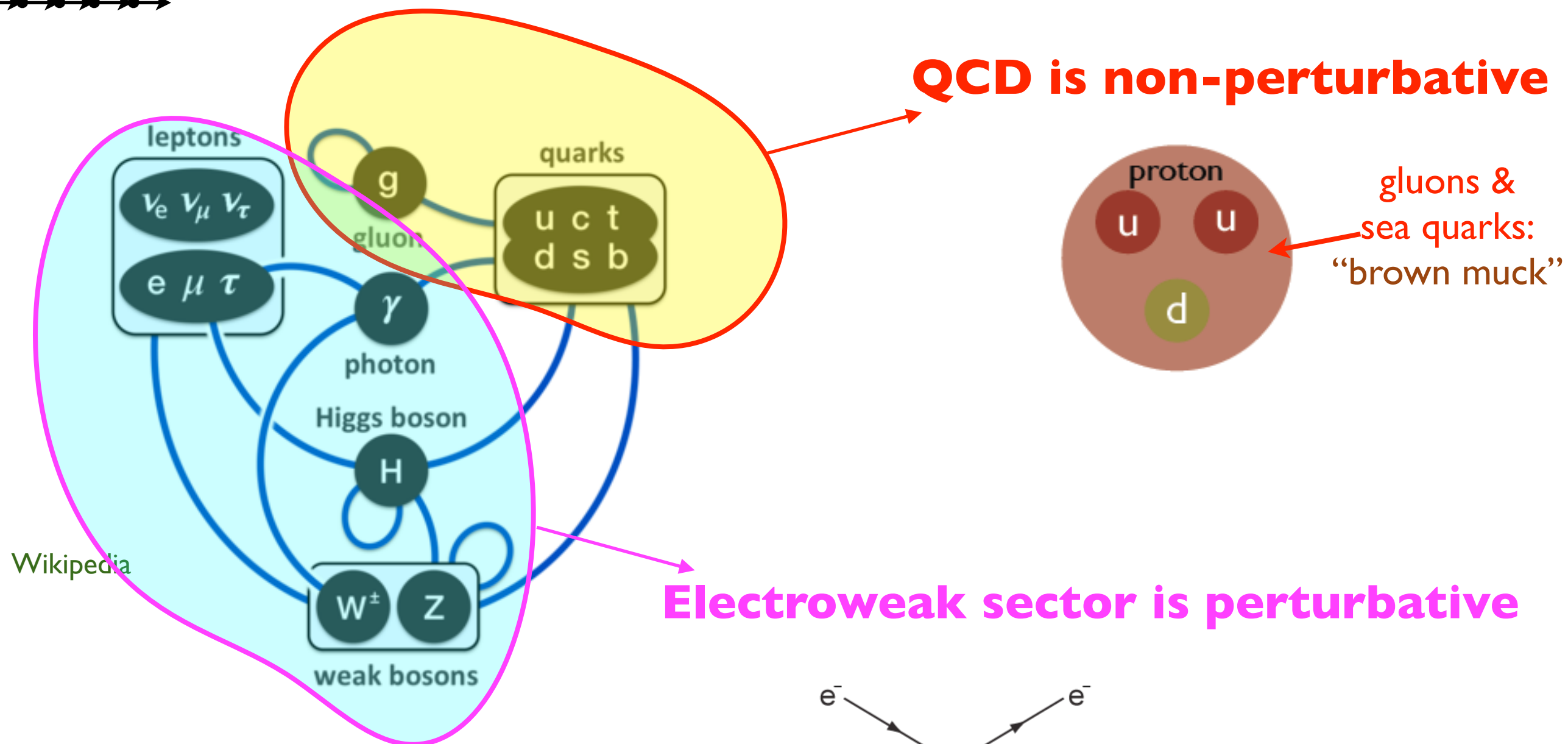
Wikipedia

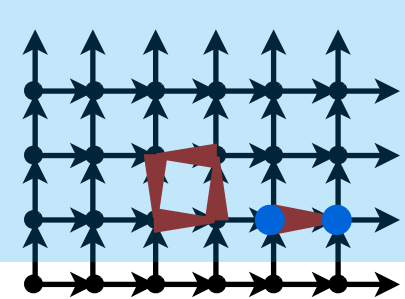
[nobelprize.org](https://www.nobelprize.org)





Standard Model (SM)





SM is extremely successful

- EM sector tested to extraordinary precision

$$\frac{4\pi}{e^2} = \frac{1}{\alpha} = 137.035999166(15)$$

Electron g-2 [Fan et al, PRL, 2022]

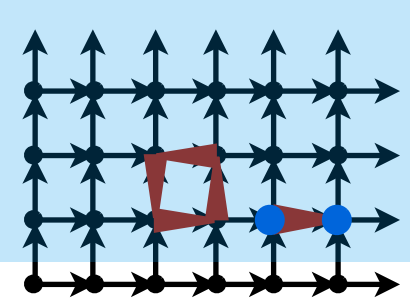
$$\frac{4\pi}{e^2} = \frac{1}{\alpha} = 137.035999206(11)$$

Rubidium recoil + Rydberg
[Morel et al, Nature, 2020]

- Weak sector tested to few parts in 1000

This is possible because couplings are weak enough to use perturbation theory

$$g_e - 2 = \frac{\alpha}{\pi} + C_2 \left(\frac{\alpha}{\pi}\right)^2 + C_3 \left(\frac{\alpha}{\pi}\right)^3 + C_4 \left(\frac{\alpha}{\pi}\right)^4 + C_5 \left(\frac{\alpha}{\pi}\right)^5 + \dots$$



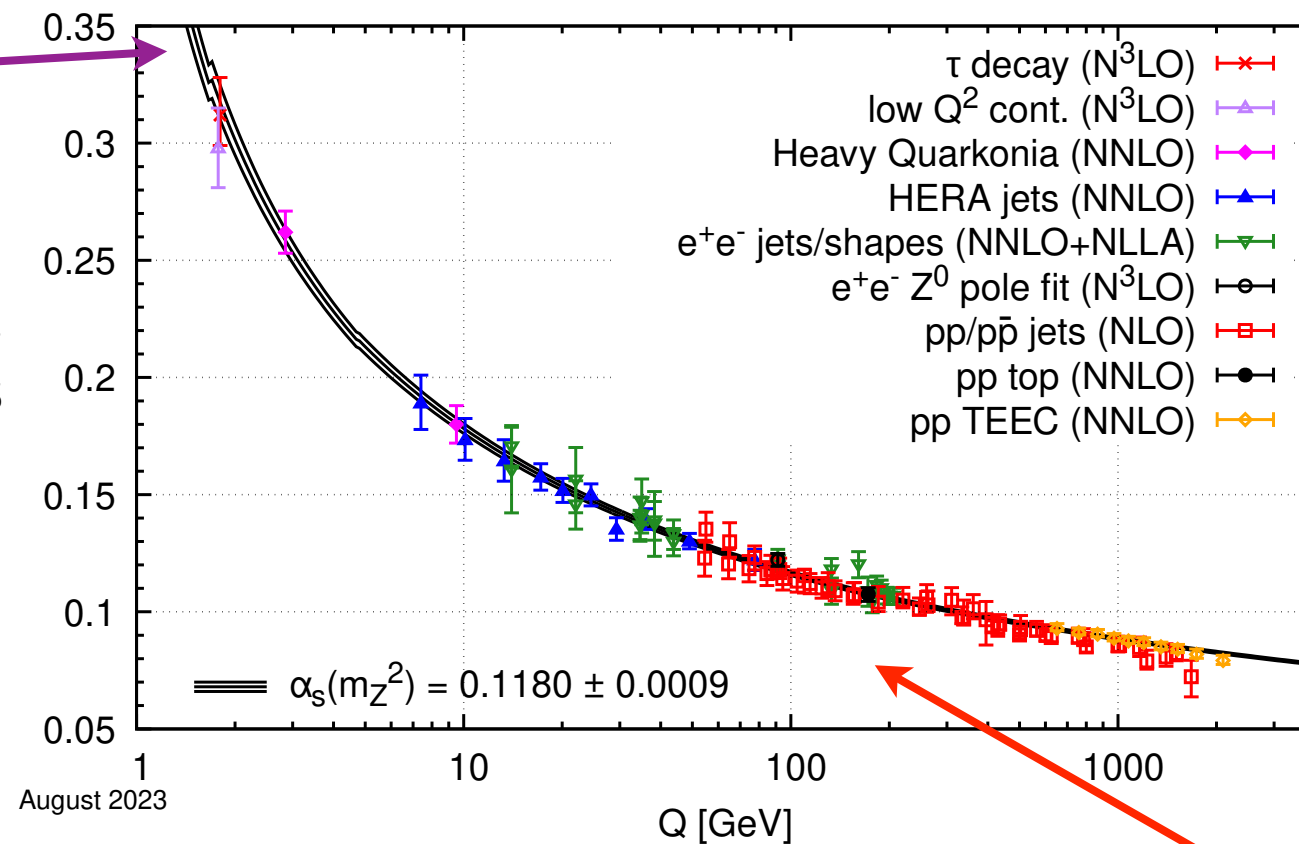
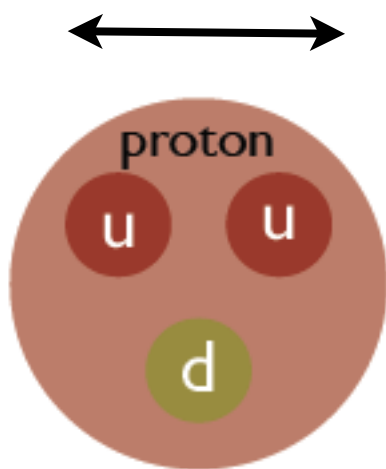
QCD is more challenging

PDG, 2023

Need non-perturbative method here

$$\frac{g^2}{4\pi} = \alpha_s(Q^2)$$

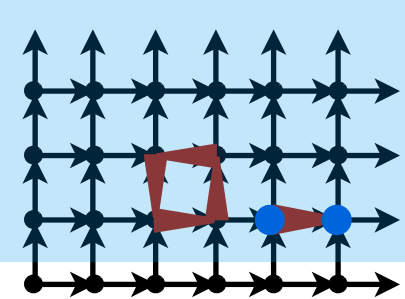
$1\text{fm} = 10^{-15}\text{m}$



$a=0.2\text{ fm}$

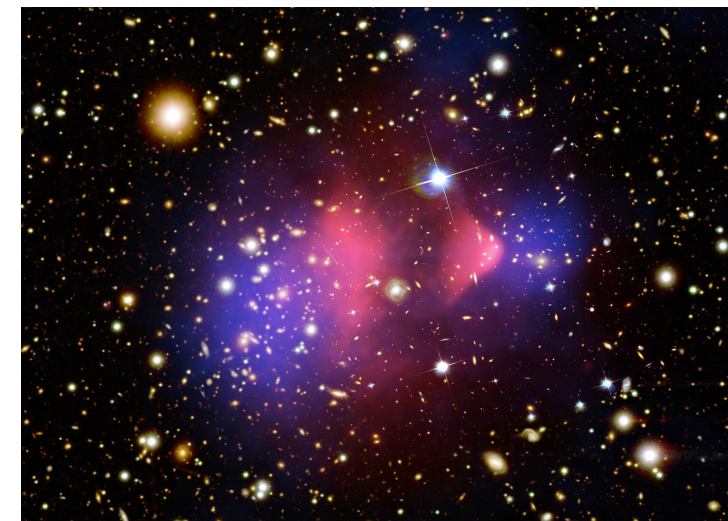
$a=0.002\text{ fm}$

Asymptotic freedom:
can use perturbation
theory at high energies

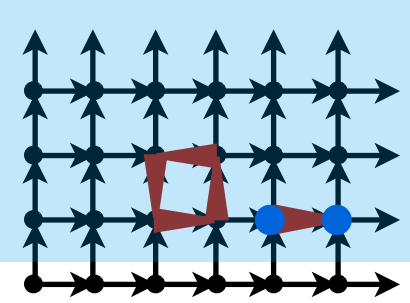


Shortcomings of the SM

- No dark matter or dark energy
- Predicts insufficient baryogenesis
- Why 3 generations? Why the observed pattern of quark & lepton masses and weak couplings?
- Weak scale relative to Planck scale
- ...

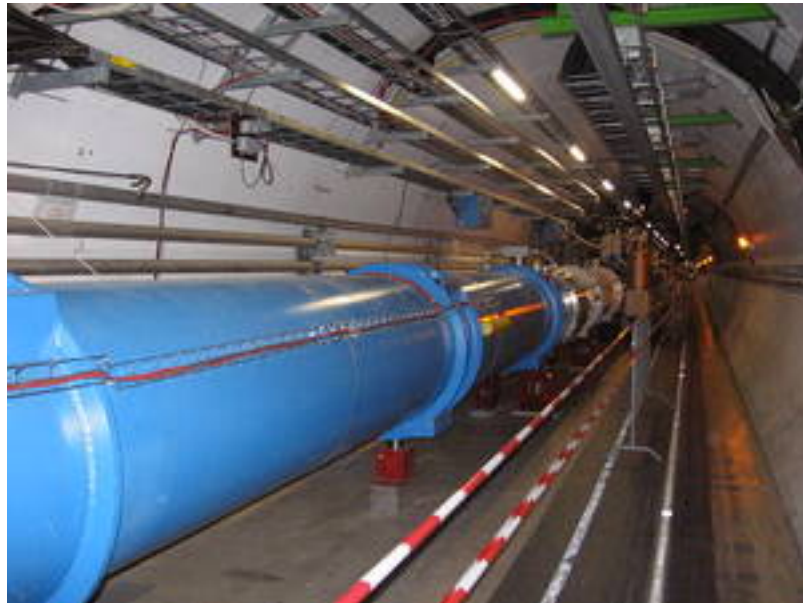


NASA



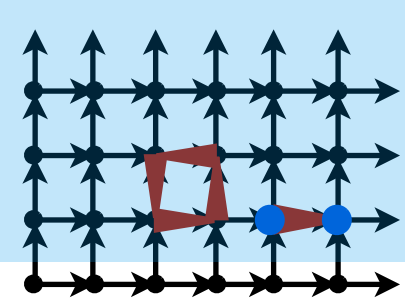
Searching for new physics

- At the highest energies—LHC



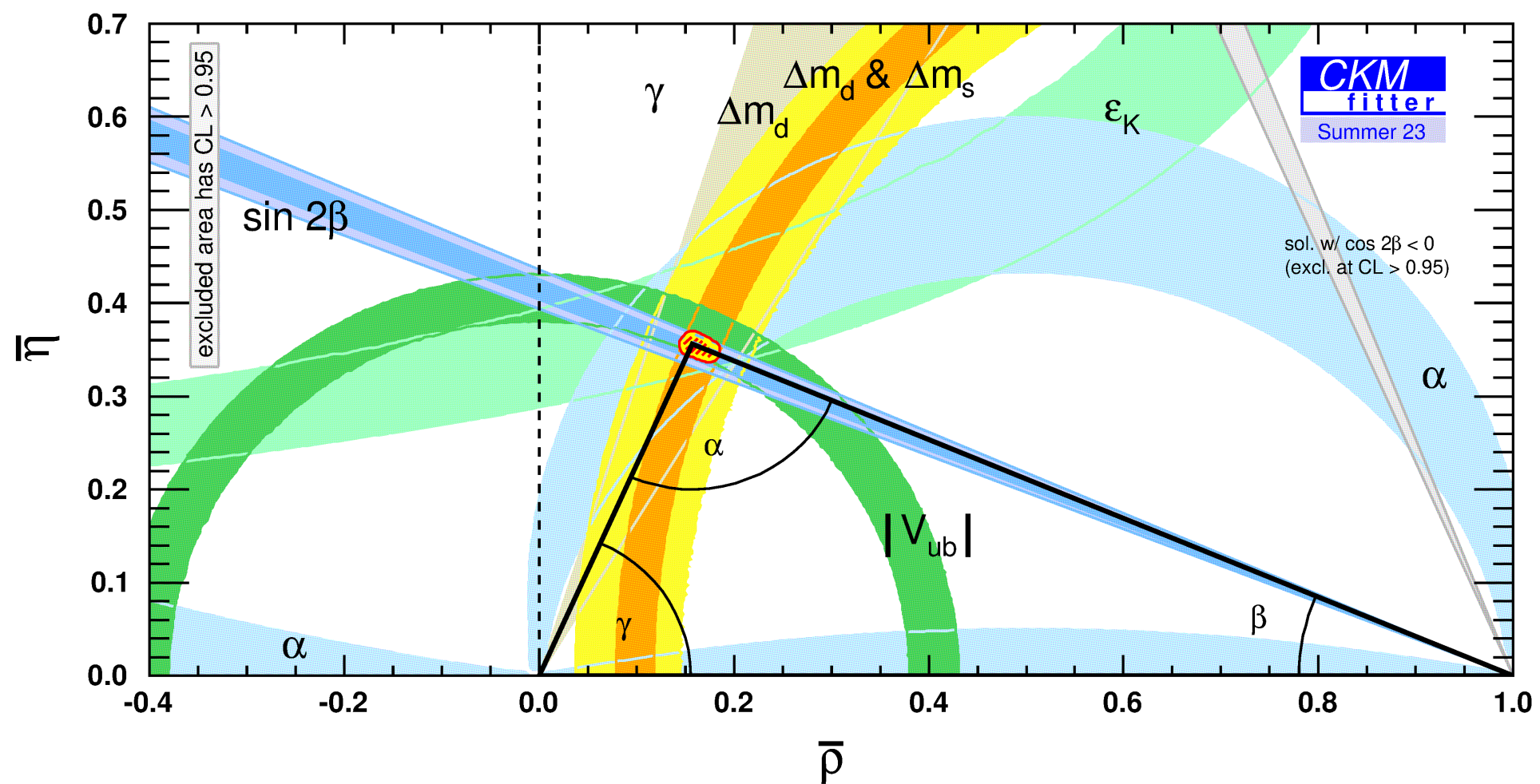
coffeeshopphysics.com

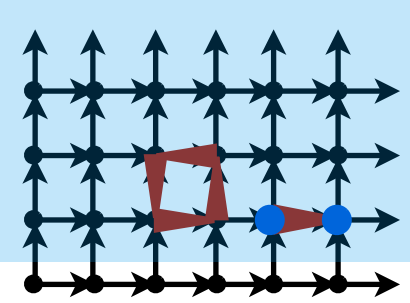




Searching for new physics

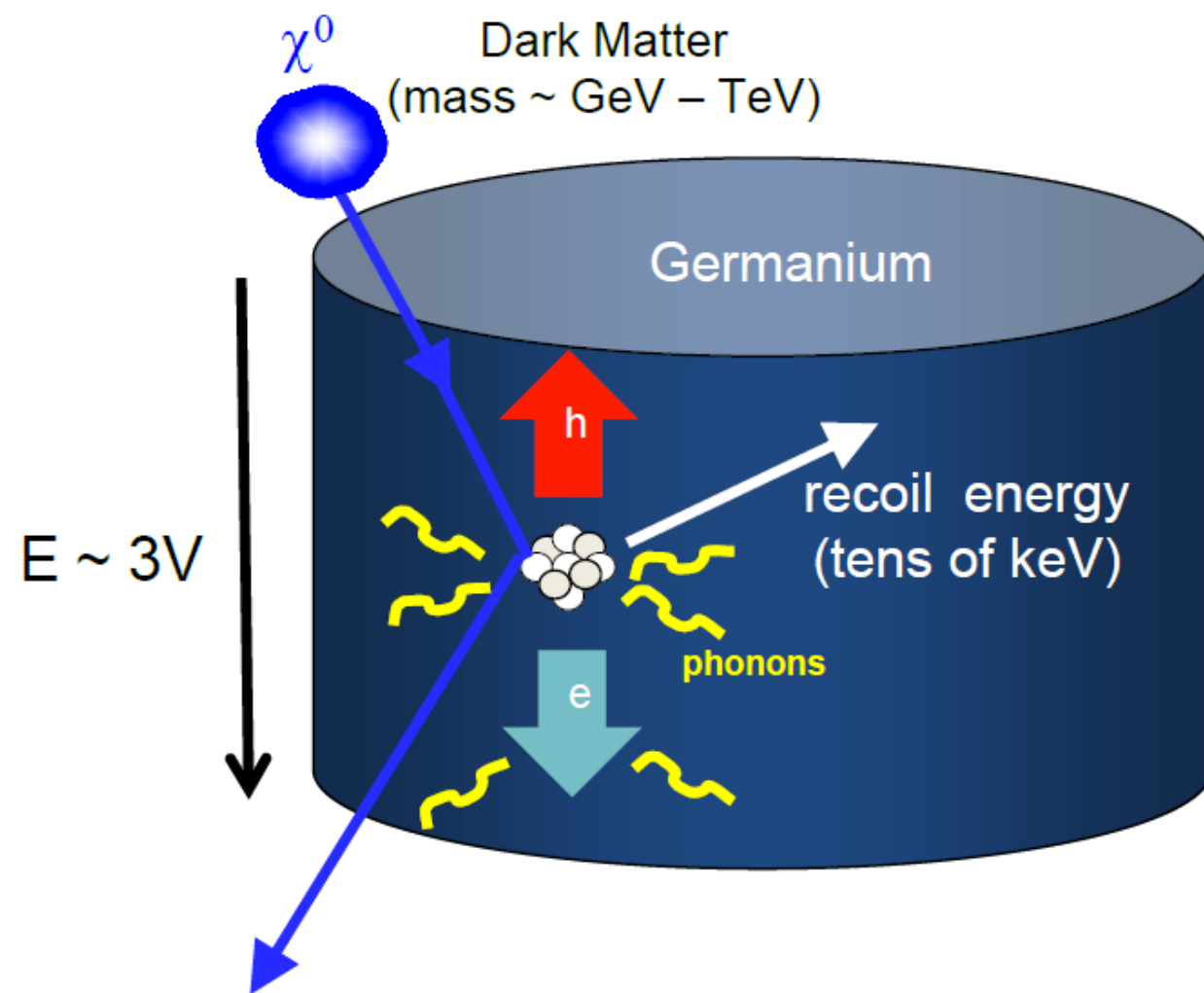
- In rare decays or precision measurements
 - Do all CP-violating processes and other precision results in K and B decays agree with the SM?



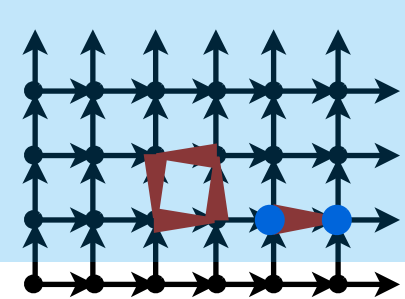


Searching for new physics

- Dark matter and axion searches

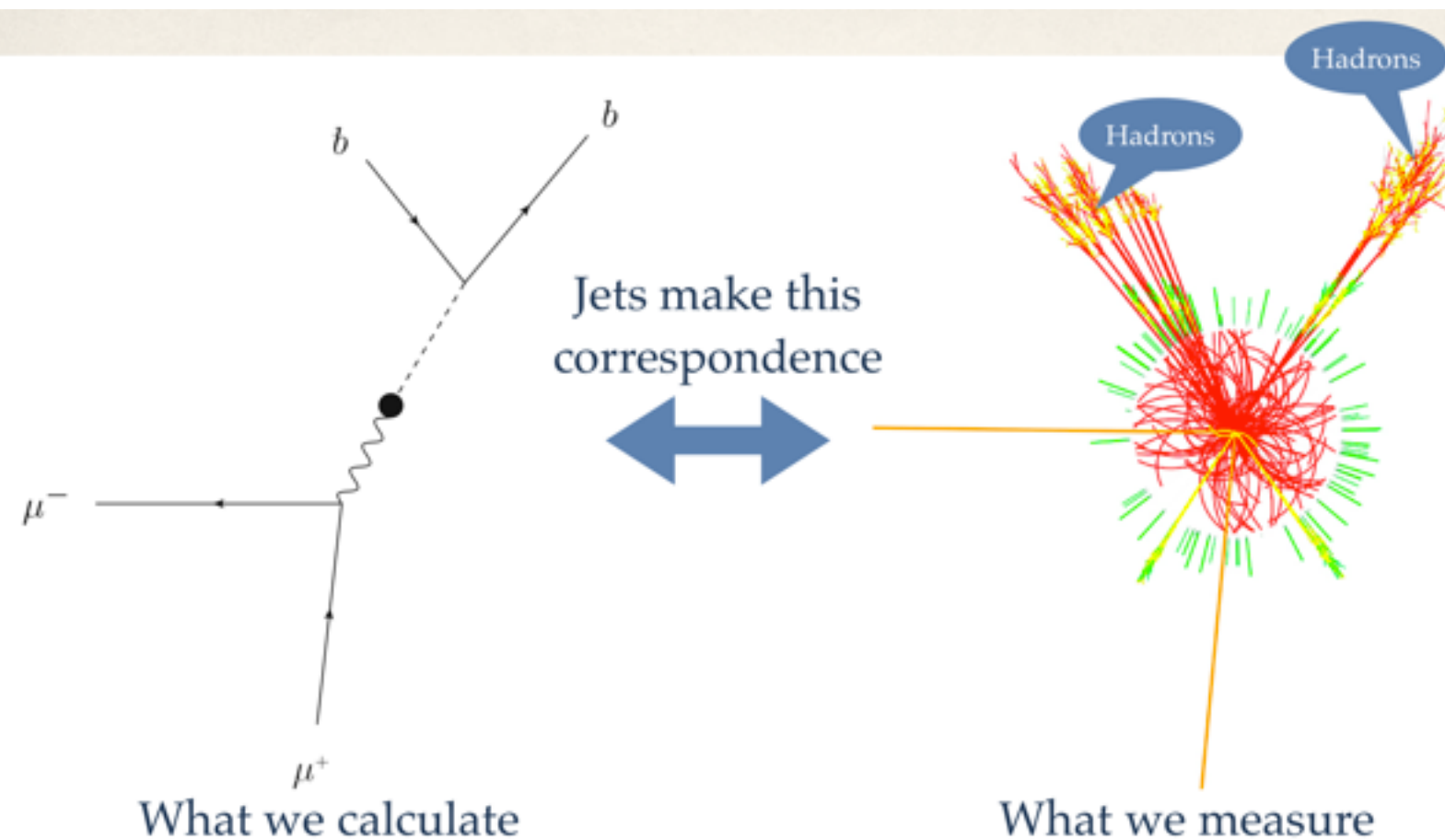


CDMS—graphic from SLAC



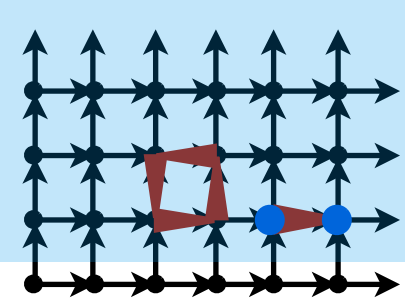
QCD as a background

- Quarks become jets at the LHC



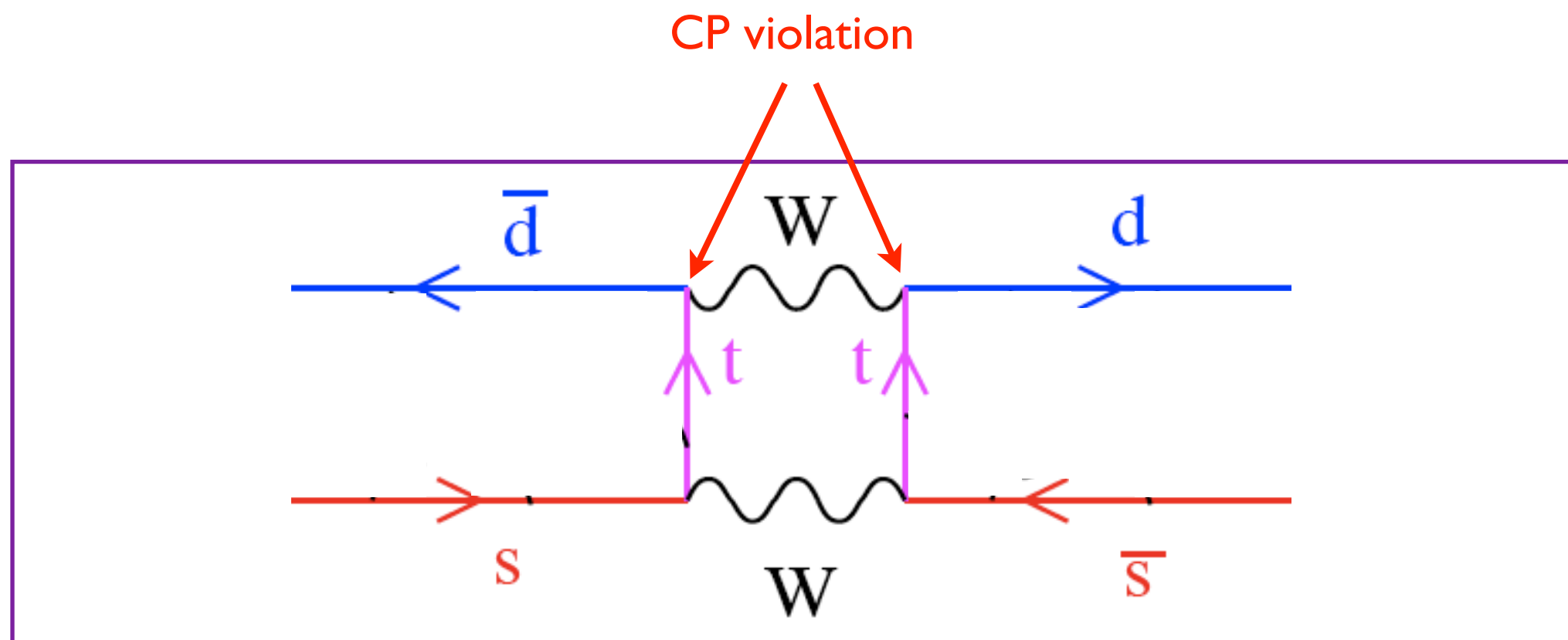
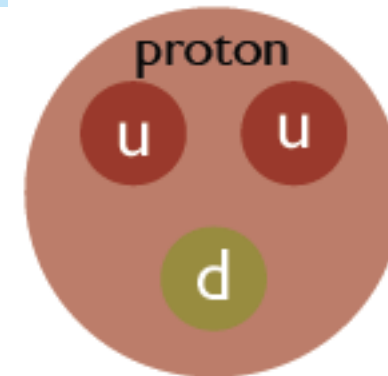
Disentangling
requires perturbative
QCD and modeling
of non-perturbative
confinement physics

quantumdiaries.org

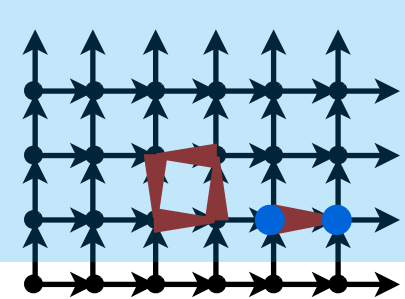


QCD as a background

- “Brown muck” distorts hadronic decays
- E.g. CP-violation in kaon-antikaon mixing

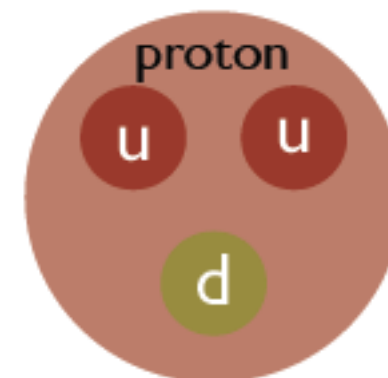


Quark level process that one might hope to
calculate in perturbation theory
is really a hadronic process that involves
non-perturbative QCD



QCD as a background

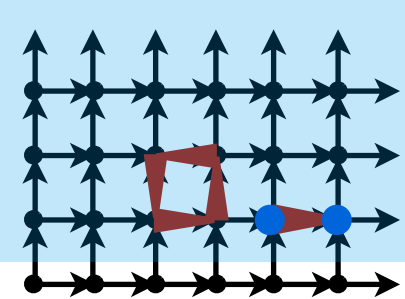
- “Brown muck” distorts hadronic decays
- Distortions can be huge, e.g. $\Delta I = 1/2$ rule



$$\frac{\Gamma(K_S^0 \rightarrow \pi\pi)}{\Gamma(K^+ \rightarrow \pi\pi)} \approx 330$$

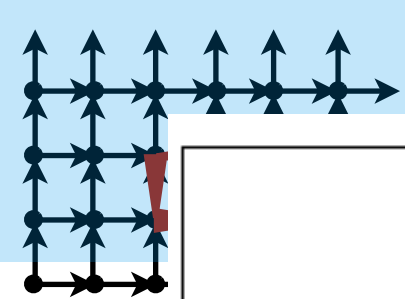
Same underlying quark weak decay: $s \longrightarrow \bar{u}ud$

- Must be able to calculate these “distortions” to interpret many rare decay experiments

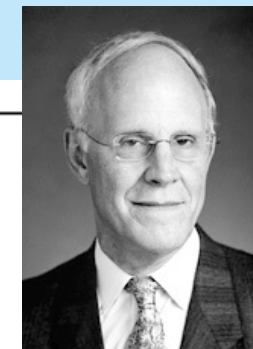


Outline

- Standard model & searching for physics beyond
- **Q**CD & Lattice **Q**CD
- High precision lattice QCD
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What is QCD?



$$\mathcal{L}_{QCD} = \bar{q}_{a,i} [(i\gamma^\mu \partial_\mu - m_i) \delta_{ab} \delta_{ij}] q_{b,j} - g G_\mu^a \bar{q}_{i,b} \gamma^\mu T_{bc}^a q_{i,c} - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

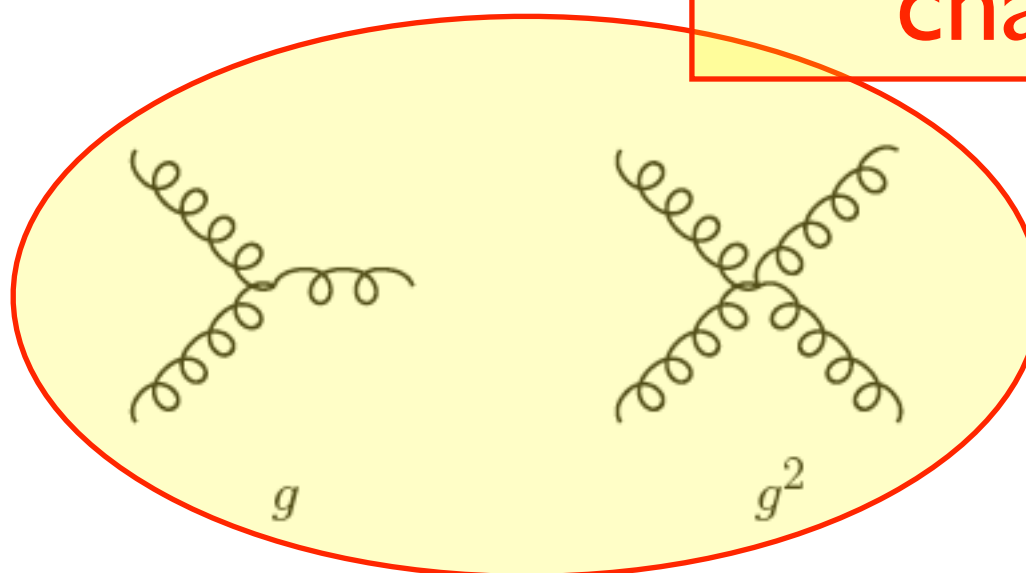
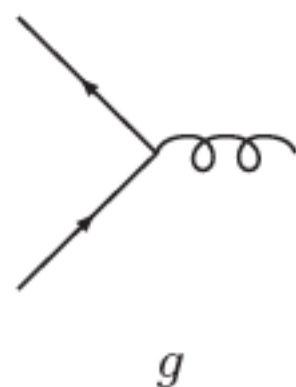
$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g f^{abc} G_\mu^b G_\nu^c$$

$q_{a,i}$ = quark, 3 colors “a” and 6 flavors “i”

G_μ^a = gluon, 8 colors “a”

g = QCD coupling

These make QCD
challenging!



Lattice QCD

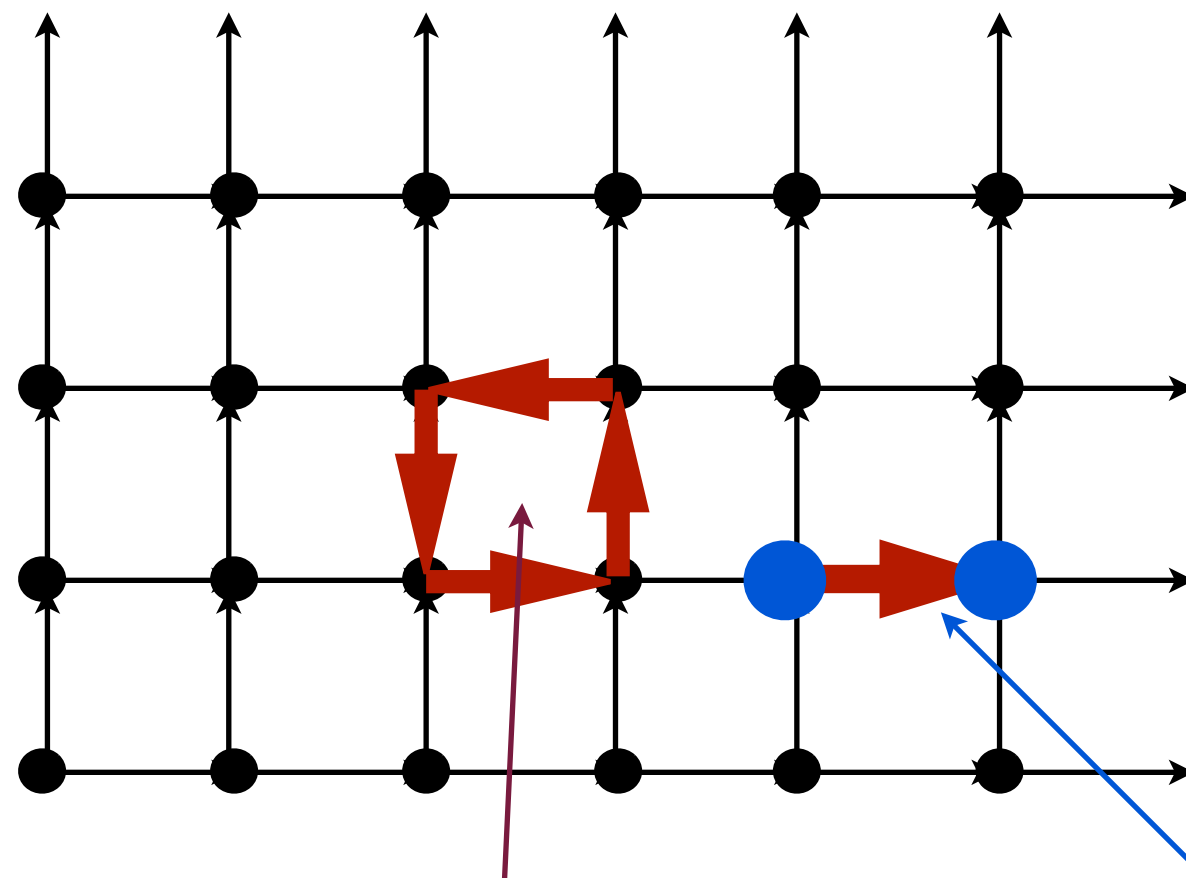
Ken Wilson, 1974



Euclidean
time



Space
3-dim



$$S_E^{\text{latt}} = - \sum_{\square} \frac{6}{g^2} \text{Re tr}_N(U_{\square, \mu\nu}) - \sum_q \bar{q}(D_{\mu}^{\text{lat}} \gamma_{\mu} + am_q)q$$

Wilson gauge action

Lattice fermion action

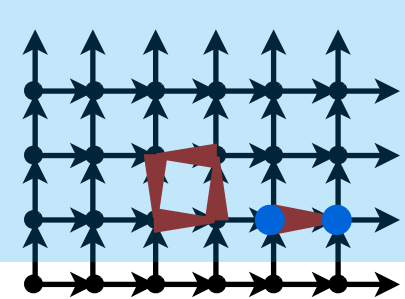
Lattice QCD



Use Feynman
path integral
definition of QM

$$Z_E = \int \prod dU d\bar{q} dq e^{-S_E^{\text{lat}}}$$

- Non-perturbative regularization of QFT
- Provides rigorous definition of QCD
 - Take $a \rightarrow 0$ by sending $g(a) \rightarrow 0$
- Amenable to numerical simulation using Monte Carlo methods

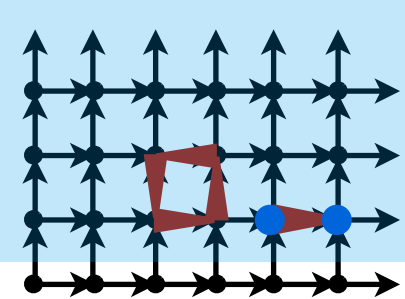


Simulating fermions is hard

$$\begin{aligned}
 Z_{\text{QCD}} &= \int \prod dU d\bar{q} dq e^{-S_E^{\text{lat}}} \\
 &= \int dU e^{-S_{\text{glue}}^{\text{lat}}} \prod_q \det (D_{\mu}^{\text{lat}} \gamma_{\mu} + m)
 \end{aligned}$$

gluon loops
fermion loops

- Fermion determinant leads to non-local effective gauge action
- Orders of magnitude more difficult to simulate than the “pure gluon” theory



Timeline

Cray I, 1 MFlop/s

- 1974, invention of lattice QCD (K.Wilson)
- 1980, simulations of pure gluon theory demonstrate confinement (M. Creutz)

Cray 2, 1 GFlop/s

- 1980's: "quenched era" (no quark loops)
- 1987, Hybrid Monte Carlo (S. Duane)
- 1990's, staggered fermions (Neuberger)
- 1990's, improved staggered fermions (Kaplan)

1 TFlop/s

Blue gene P,
1 PFlop/s

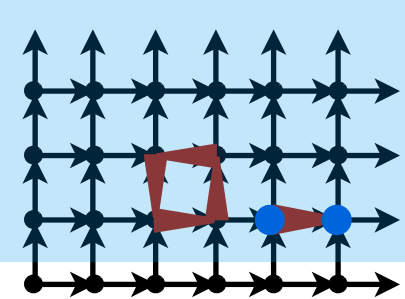
Blue gene Q,
10 PFlop/s

CPU speedup, theoretical & algorithmic advances have allowed lattice QCD to become a precision tool

- 2000's: fully unquenched era (light quark loops)
- 2009-10: simulations with physical up, down and strange quark masses
- 2010's: inclusion of electromagnetism, isospin breaking
- 2020's: many extensions: inclusive processes, multi particle scattering

Summit, 100 PFlop/s

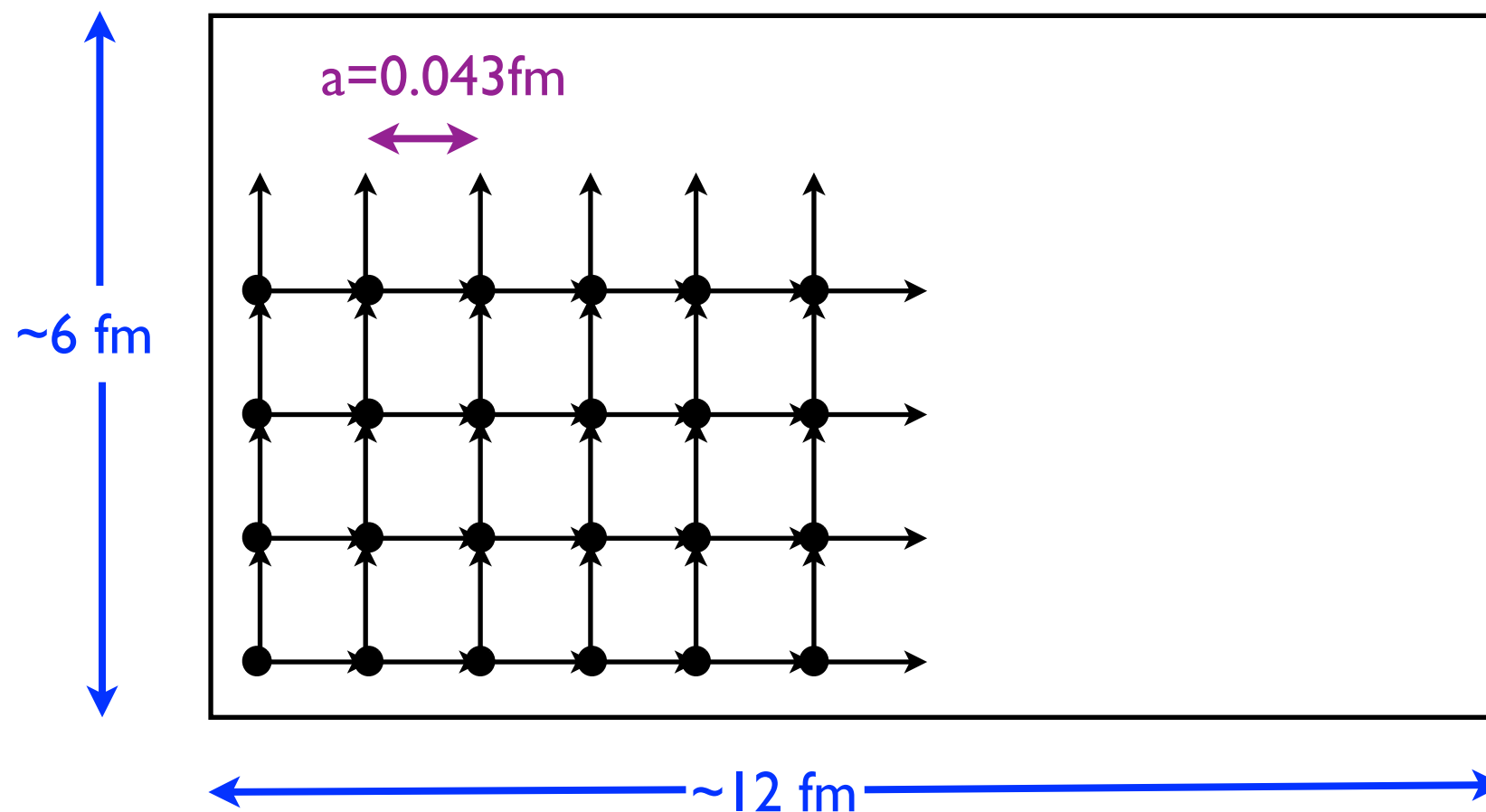
El Capitan, > 1 ExaFlop/s



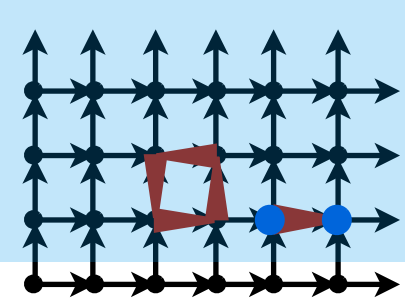
State of the art

144x144x144x288 lattice [MILC collaboration]

Highly Improved Staggered (HISQ) fermions
Physical quark masses (in isospin limit: $m_u = m_d$)



Need to invert matrices
of size
 $\sim (3 \times 10^9) \times (3 \times 10^9)$



State of the art

Extrapolating to the physical point

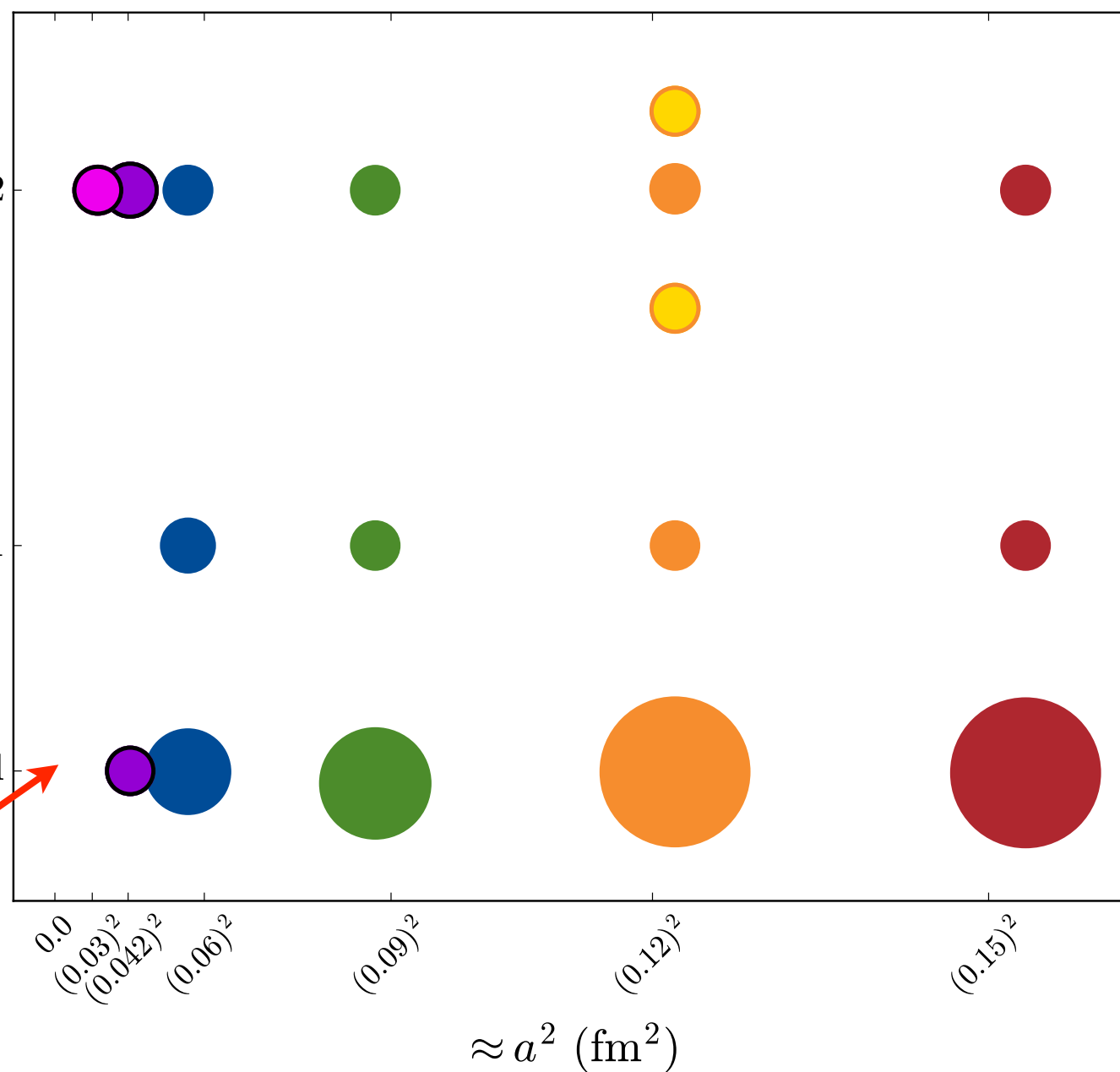
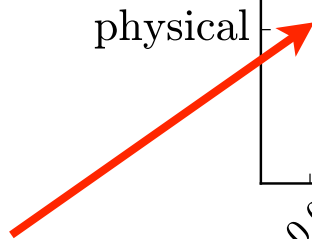
Decreasing u,d
quark masses



$$\frac{m_u + m_d}{2m_s}$$

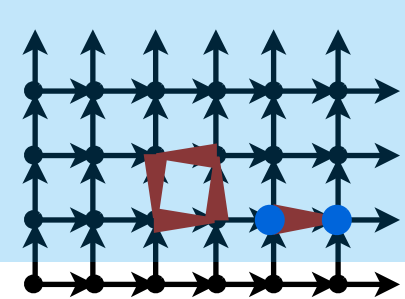
physical

physical point



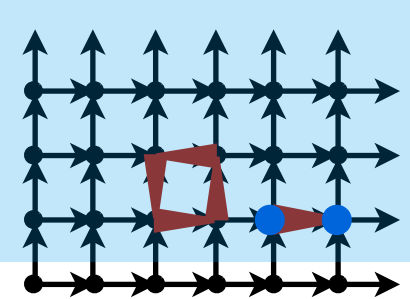
continuum limit

S. Gottlieb,
talk at NERSC@50, 2025



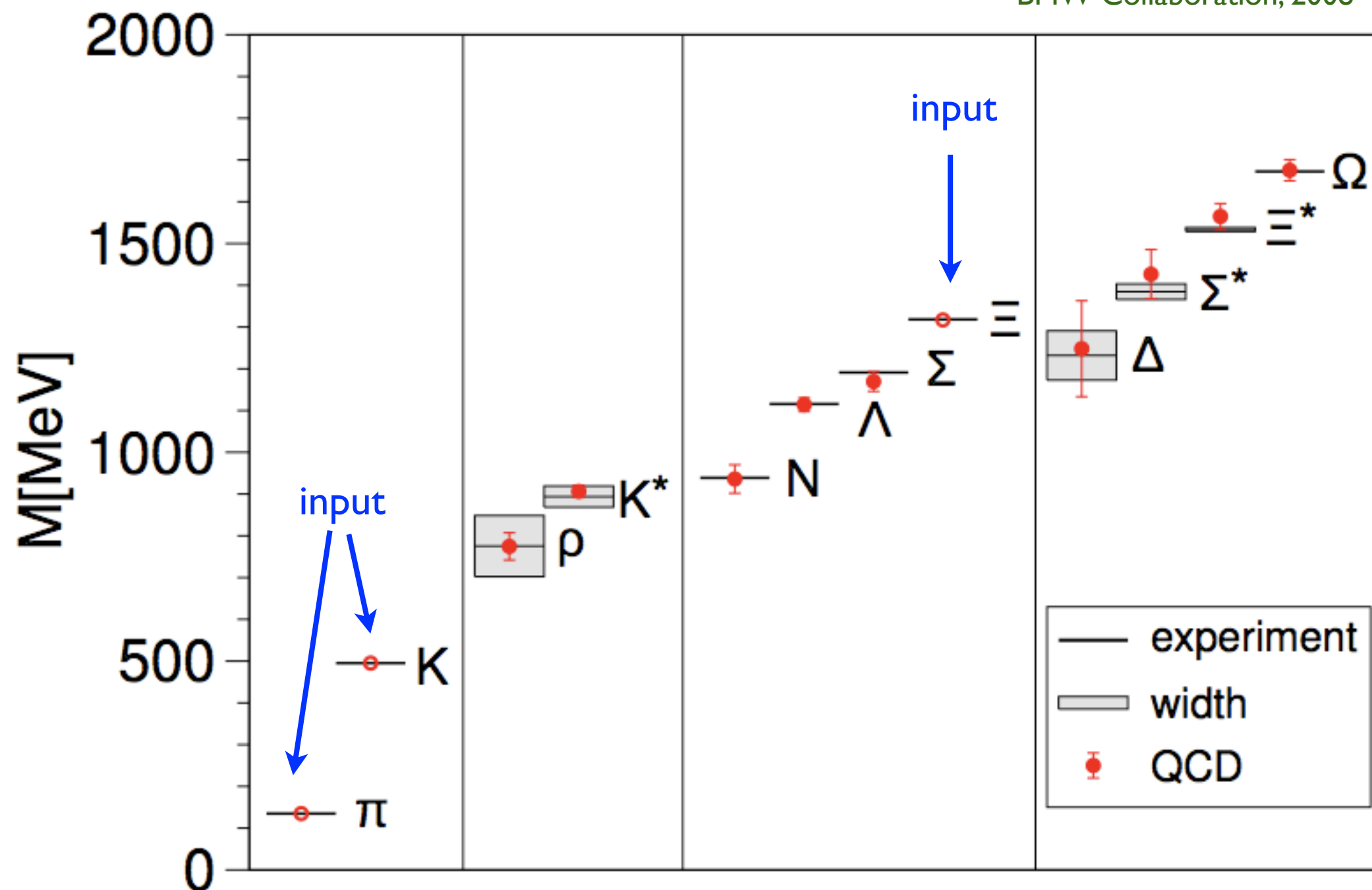
Outline

- Standard model & searching for physics beyond
- QCD & Lattice QCD
- High precision lattice **QCD**
- Constraining the Standard Model with LQCD
- Extending the LQCD frontier
- Summary



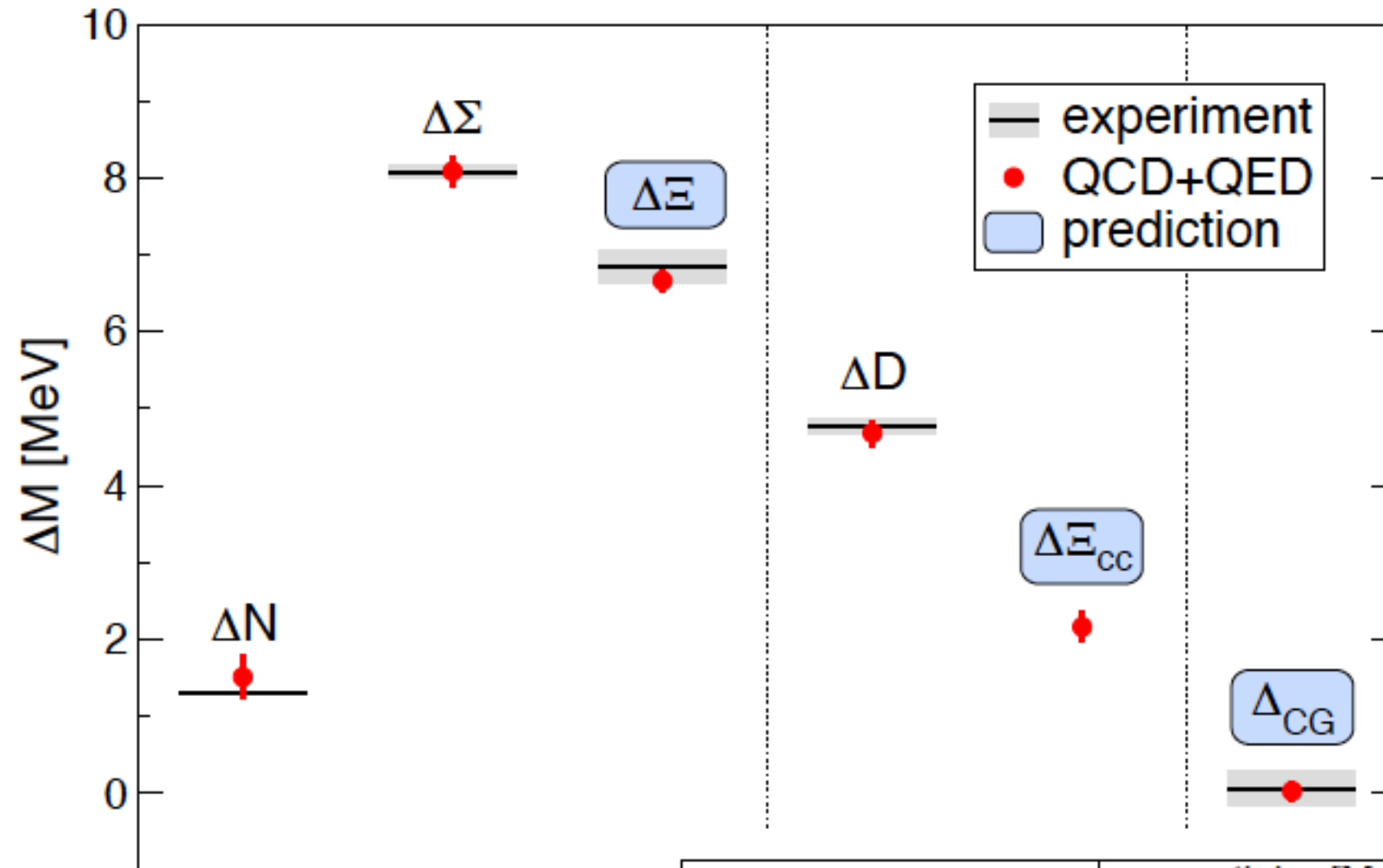
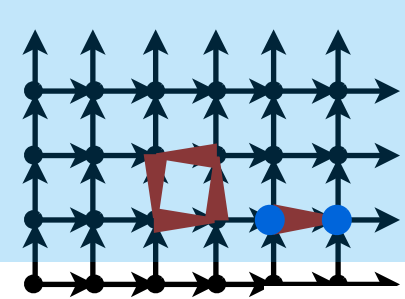
Postdiction: spectrum

BMW Collaboration, 2008



Few percent accuracy, and complete consistency

Isospin splittings



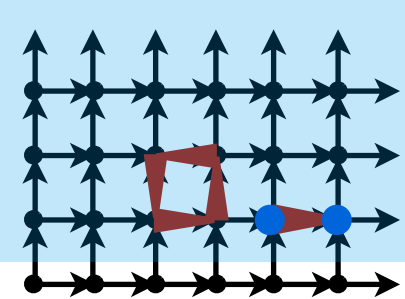
BMW Collaboration
2014

u, d, s & c in loops
 $m_u \neq m_d$
QED included

quark masses & scale
determined using
 π^+ , K^+ , K^0 , D^0 , Ω

Errors ~ 0.2 MeV !

	mass splitting [MeV]	QCD [MeV]	QED [MeV]
$\Delta N = n - p$	1.51(16)(23)	2.52(17)(24)	-1.00(07)(14)
$\Delta \Sigma = \Sigma^- - \Sigma^+$	8.09(16)(11)	8.09(16)(11)	0
$\Delta \Xi = \Xi^- - \Xi^0$	6.66(11)(09)	5.53(17)(17)	1.14(16)(09)
$\Delta D = D^\pm - D^0$	4.68(10)(13)	2.54(08)(10)	2.14(11)(07)
$\Delta \Xi_{cc} = \Xi_{cc}^{++} - \Xi_{cc}^+$	2.16(11)(17)	-2.53(11)(06)	4.69(10)(17)
$\Delta_{CG} = \Delta N - \Delta \Sigma + \Delta \Xi$	0.00(11)(06)	-0.00(13)(05)	0.00(06)(02)



Flavo(u)r Lattice Averaging Group

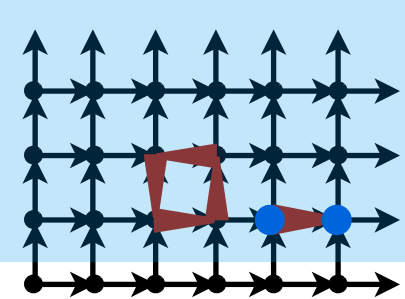
arXiv:2411.04268v2 [hep-lat] 17 Jan 2025

FLAG Review 2024 (6th edition)

Flavour Lattice Averaging Group (FLAG)

Y. Aoki¹, T. Blum^{2,3}, S. Collins⁴, L. Del Debbio⁵, M. Della Morte⁶, P. Dimopoulos^{7,8},
X. Feng^{9,10,11,12}, M. Golterman¹³, Steven Gottlieb¹⁴, R. Gupta¹⁵, G. Herdoiza¹⁶,
P. Hernandez¹⁷, A. Jüttner^{18,19,20}, T. Kaneko^{21,22}, E. Lunghi¹⁴, S. Meinel²³,
C. Monahan^{24,25}, A. Nicholson²⁶, T. Onogi²⁷, P. Petreczky²⁸, A. Portelli^{1,5,20},
A. Ramos¹⁷, S. R. Sharpe²⁹, J. N. Simone³⁰, S. Sint³¹, R. Sommer^{32,33}, N. Tantalo³⁴,
R. Van de Water³⁰, A. Vaquero^{35,36}, U. Wenger³⁷, and H. Wittig^{38,39}

- Reviews every ~3 years: “vetted” averages of 72 **LQCD** results with full error budgets
- “PDG (Particle Data Group) or HFLAV (Heavy Flavor Working Group) for Lattice QCD”



FLAG6: QCD parameters

- Strong coupling constant

$$\alpha_{\overline{\text{MS}}}^{(5)}(M_Z) = 0.1183(7) \quad [\text{0.6\% error}]$$

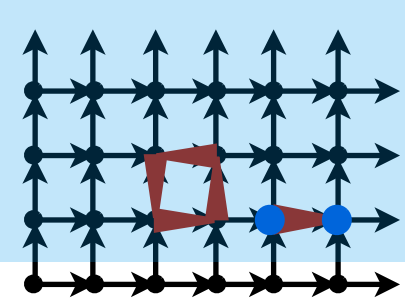
Compare to

$$\alpha_s(m_Z^2) = 0.1175 \pm 0.0010 \quad (\text{PDG 2023 without lattice})$$

- Quark masses

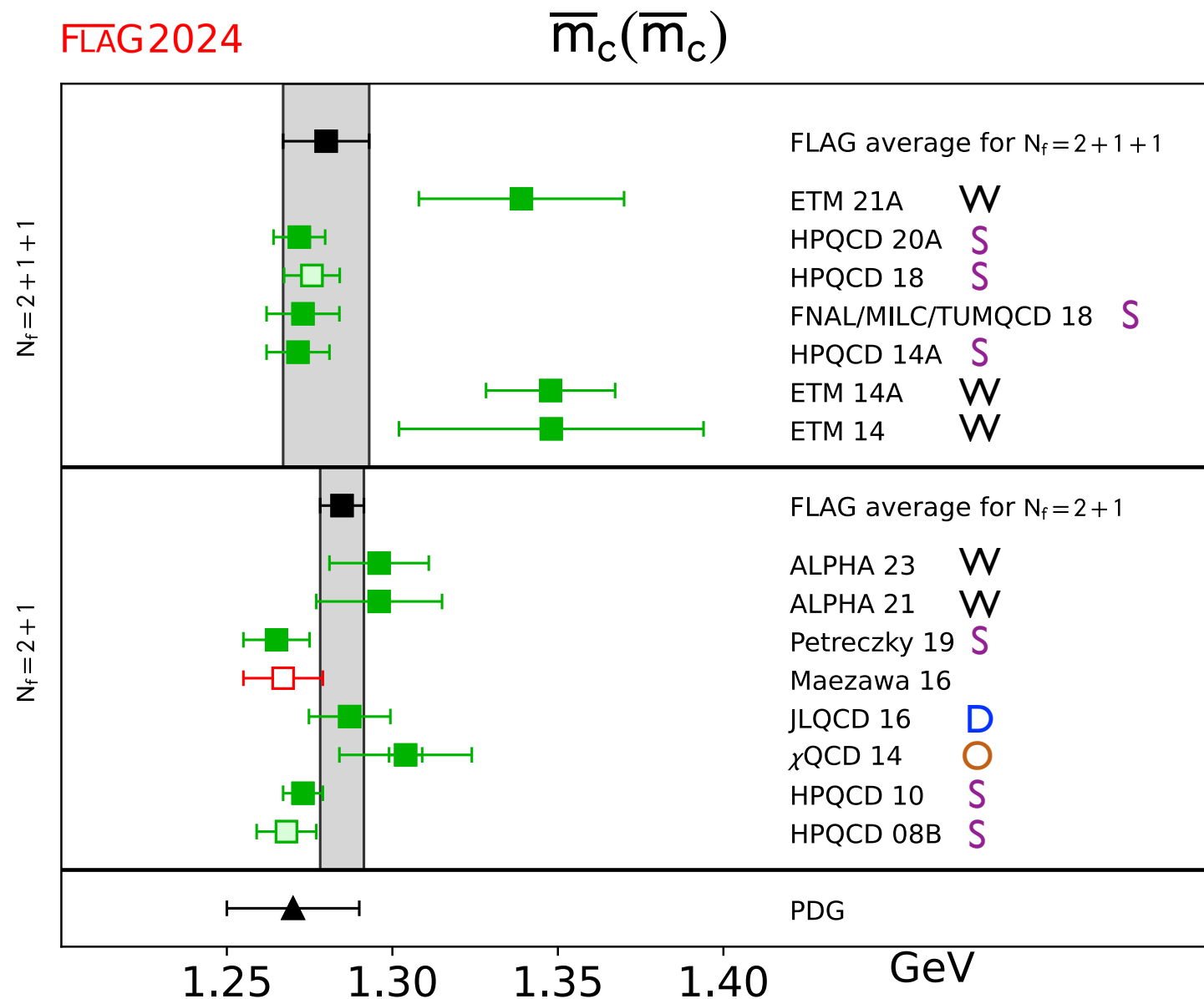
$$m_u[\text{4\%}], m_d[\text{2\%}], m_s[\text{1\%}], m_c[\text{1\%}], m_b[\text{0.3\%}]$$

$$\frac{m_u + m_d}{m_s}[\text{0.3\%}], \frac{m_s}{m_c}[\text{0.3\%}]$$



Example: charm quark mass

$$\bar{m}_c(\bar{m}_c) = 1.278(6) \text{ GeV } [N_f = 2 + 1]$$

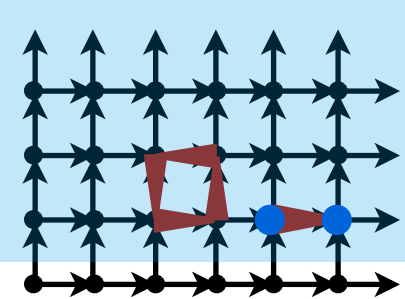


W=Wilson fermions

S=staggered fermions

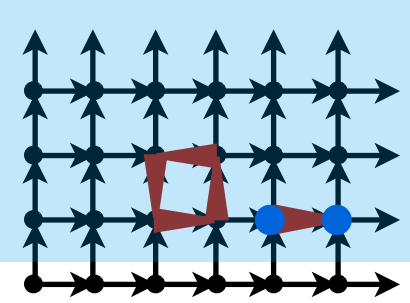
D=domain-wall fermions

O=overlap fermions



Outline

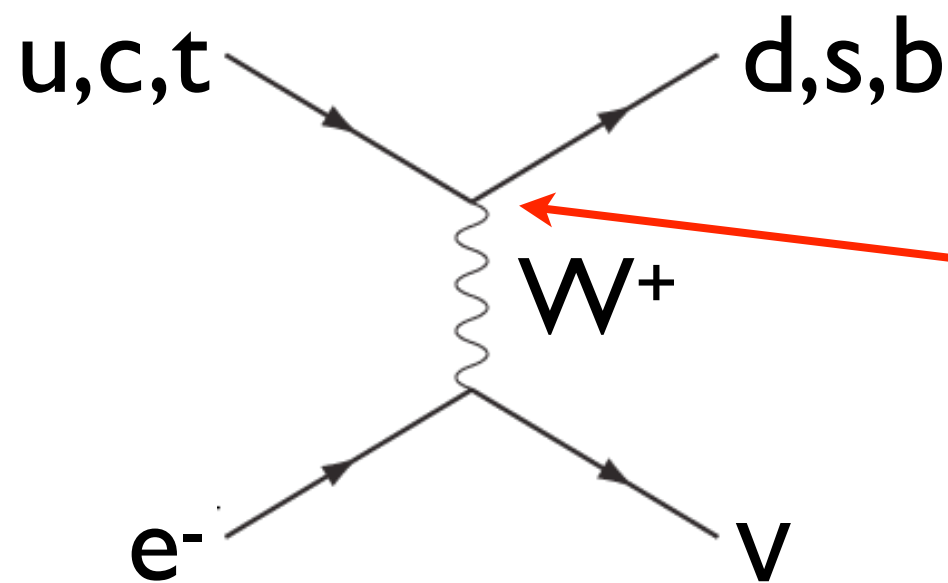
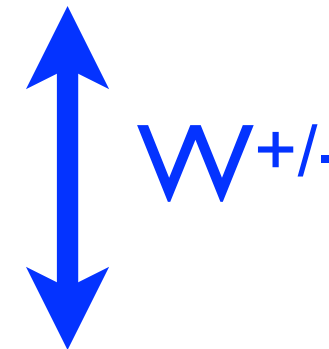
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CKM matrix & CP violation

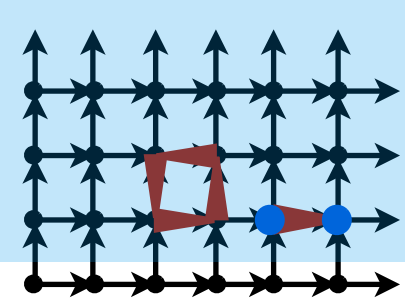
Three Generations of Matter (Fermions)

	I	II	III
mass →	2.4 MeV	1.27 GeV	171.2 GeV
charge →	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
spin →	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
name →	u up	c charm	t top
	4.8 MeV	104 MeV	4.2 GeV
	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
Quarks	d down	s strange	b bottom



$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Fundamental parameters of the SM



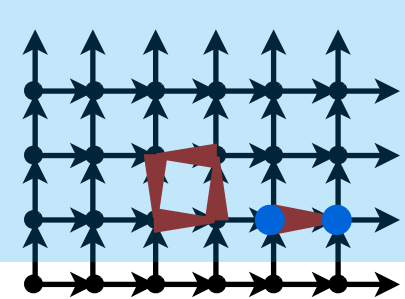
CKM matrix & CP violation

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}.$$

Unitary matrix

CP violation!

- Each element can be measured in several ways
- Consistency of SM requires all measurements to agree, and that V_{CKM} be unitary
- The CP violating parameter η must explain observed CP violation in Kaon and B meson (and D) systems
- New physics would show up as inconsistencies



Need for non-perturbative QCD

$$\begin{pmatrix}
 \mathbf{V}_{ud} & \mathbf{V}_{us} & \mathbf{V}_{ub} \\
 \pi \rightarrow \ell \nu & K \rightarrow \ell \nu & B \rightarrow \pi \ell \nu \\
 & K \rightarrow \pi \ell \nu & \\
 \mathbf{V}_{cd} & \mathbf{V}_{cs} & \mathbf{V}_{cb} \\
 D \rightarrow \ell \nu & D_s \rightarrow \ell \nu & B \rightarrow D \ell \nu \\
 D \rightarrow \pi \ell \nu & D \rightarrow K \ell \nu & B \rightarrow D^* \ell \nu \\
 \mathbf{V}_{td} & \mathbf{V}_{ts} & \mathbf{V}_{tb} \\
 B_d \leftrightarrow \bar{B}_d & B_s \leftrightarrow \bar{B}_s & \\
 K_0 \leftrightarrow \bar{K}_0 & K_0 \leftrightarrow \bar{K}_0 &
 \end{pmatrix}$$

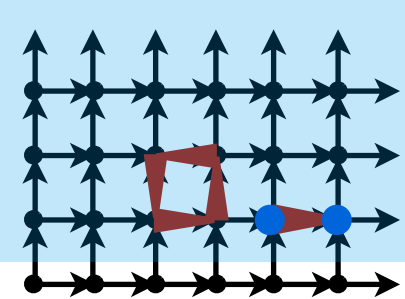
Experiment = known factors $\times V_{CKM} \times$ Hadronic matrix element

Measure $\uparrow$

Perturbative SM $\uparrow$

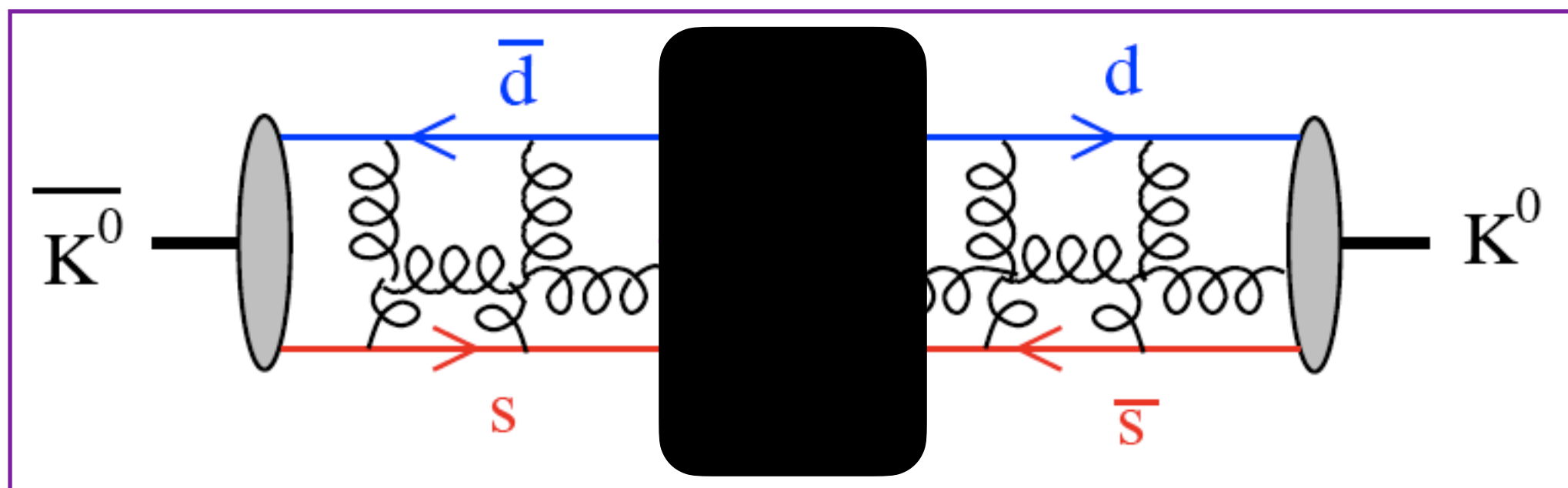
to determine this $\uparrow$

Lattice QCD $\uparrow$



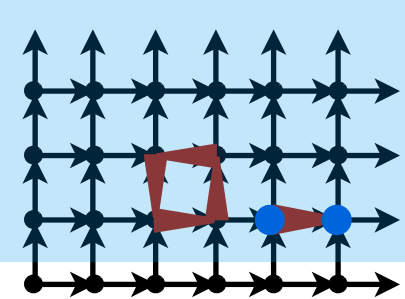
Example of hadronic matrix

- $\hat{B}_K$: CP-violation in kaon-antikaon mixing



Integrate out W (and Z) and t (and c) to obtain local operator

$$\hat{B}_K \sim \frac{\langle \bar{K}_0 | [\bar{s} \gamma_\mu (1 - \gamma_5) d] [\bar{s} \gamma^\mu (1 - \gamma_5) d] | K_0 \rangle}{\frac{8}{3} f_K^2 m_K^2}$$



FLAG6: matrix elements

- Decay constants, e.g. $\langle \pi^+ | \bar{u} \gamma^\mu \gamma_5 d | 0 \rangle \propto p_\pi^\mu f_\pi$

$$f_\pi[0.6\%], f_K[0.2\%], f_D[0.3\%], f_{D_s}[0.2\%], f_B[0.7\%] f_{B_s}[0.5\%]$$

- B-parameters, e.g. $\langle \bar{K}_0 | [\bar{s} \gamma_{\mu,L} d] [\bar{s} \gamma_L^\mu d] | K_0 \rangle \sim B_K$

$$B_K[1\%], B_B[5\%], B_{B_s}[5\%], B_{\text{BSM}}[\lesssim 3\%]$$

- Form factors, e.g. $\langle K^-(p) | \bar{s} \gamma^\mu u | \pi^+(k) \rangle$

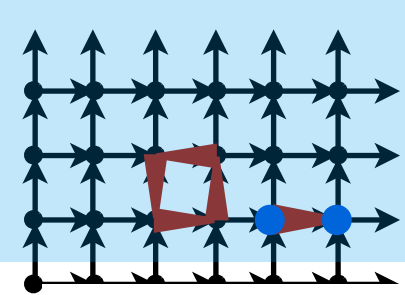
$$f_+^{K \rightarrow \pi}(0)[0.2\%], f_+^{D \rightarrow \pi}(0)[1\%], f_+^{D \rightarrow K}(0)[0.3\%]$$

Aida will
discuss some
of these

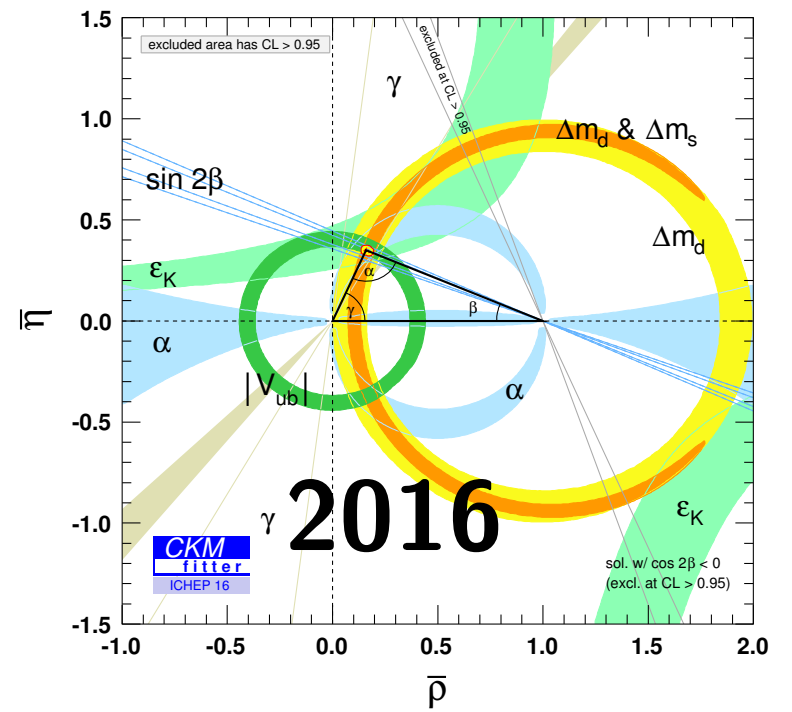
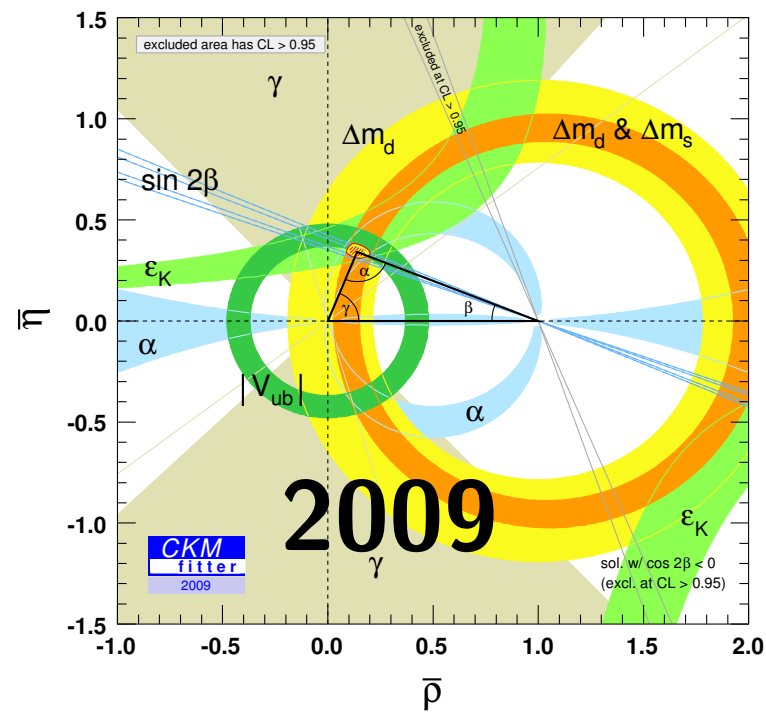
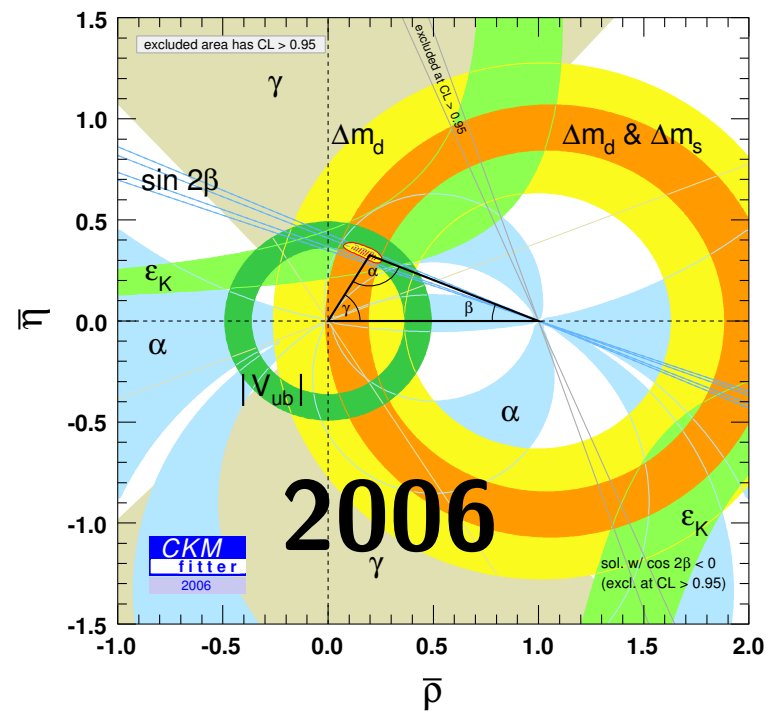
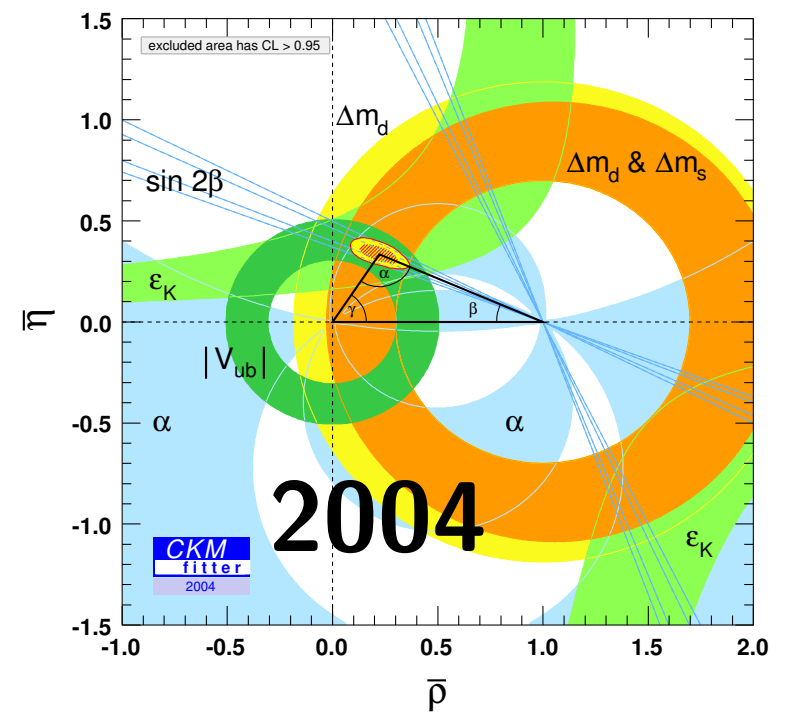
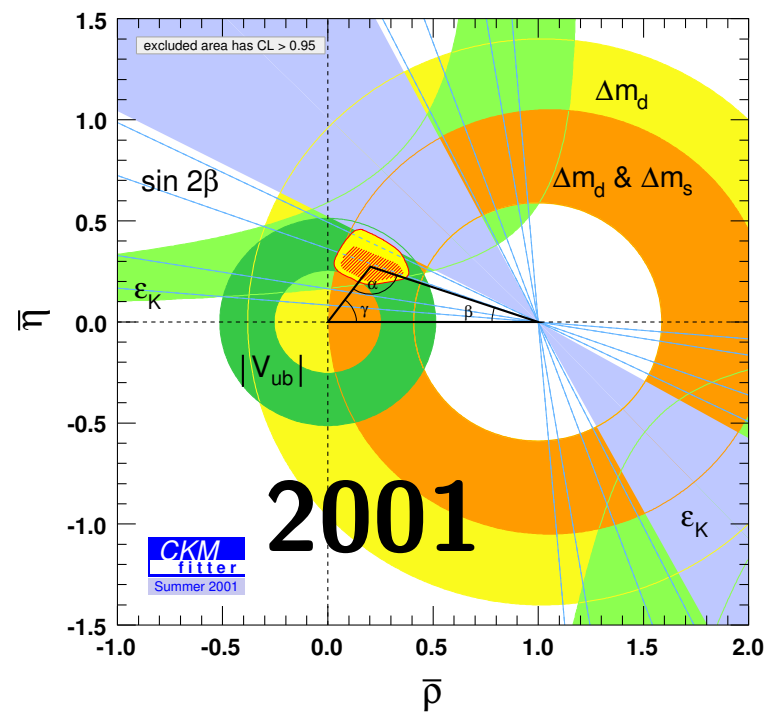
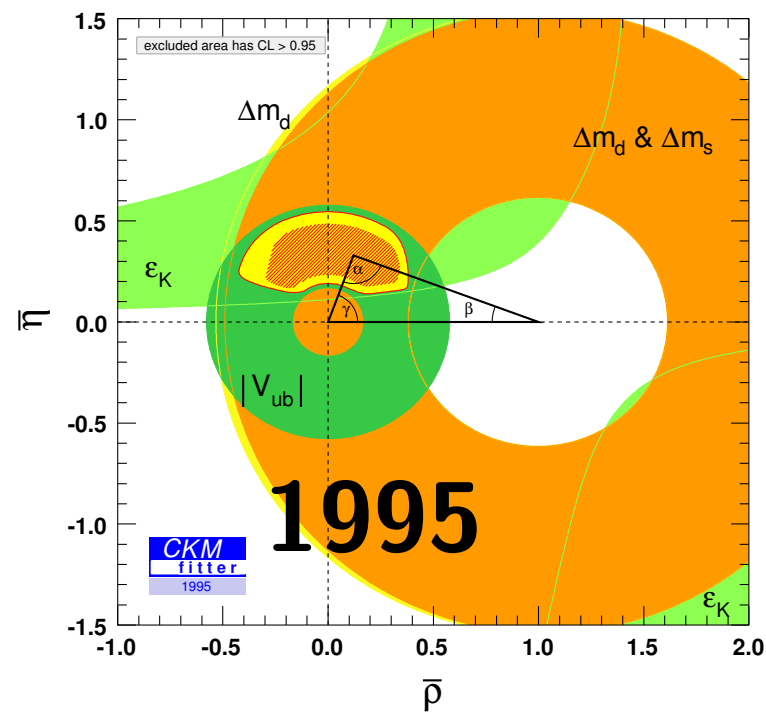
$$B \rightarrow \pi \ell \nu, B_s \rightarrow K \ell \nu, B \rightarrow D \ell \nu, B \rightarrow D^* \ell \nu, B \rightarrow D_s \ell \nu, B \rightarrow K \ell^+ \ell^-, \dots$$

- Nucleon matrix elements, e.g. $\langle N | \bar{s} s | N \rangle \propto \sigma_s$

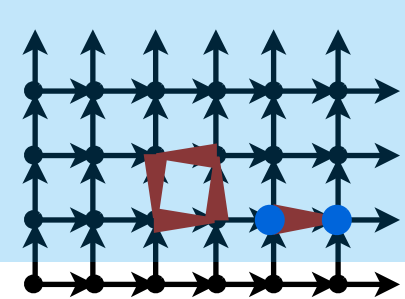
$$g_A^{u-d}[1\%], \sigma_{\pi N}[5\%], \sigma_s[15\%], \langle x \rangle_{u-d} [10\%], \dots$$



History of steady progress

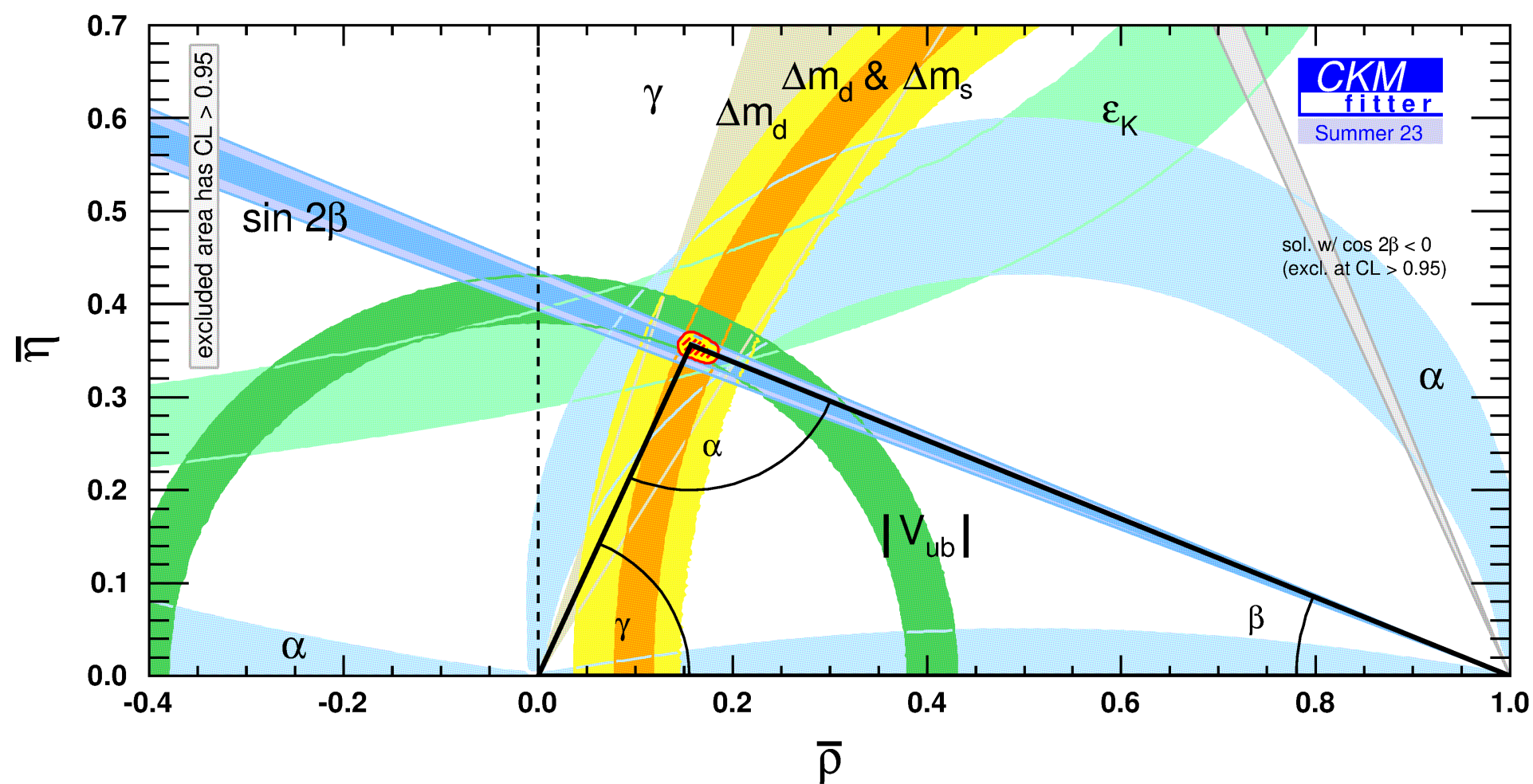


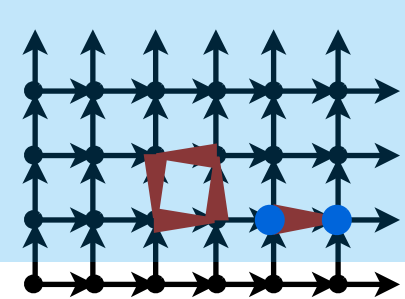
CKMfitter (ckmfitter.in2p3.fr): talk by Luiz Silva, 9/2018



...but overall consistency

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}.$$





...though this is controversial

2025 update on ε_K in the Standard Model with lattice QCD inputs

Seungyeob Jwa,¹ Jeehun Kim,¹ Sunghee Kim,¹ Sunkyu Lee,² Weonjong Lee,^{1,*} and Sungwoo Park³
(SWME Collaboration)

arXiv:2503.00351v3 [hep-lat] 13 Mar 2025

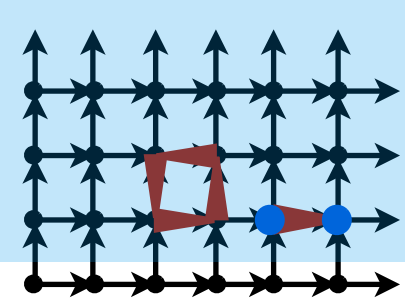
TABLE XXIV. Results for $\Delta\varepsilon_K$. They are obtained using the RBC-UKQCD estimate for ξ_{LD} , the indirect method for ξ_0 , the traditional method for η_i of $c-t$ unitarity, the FLAG results for $\hat{B}_K$, the AOF results for the Wolfenstein parameters, and so on.

year	Inclusive $ V_{cb} $	Exclusive $ V_{cb} $
2015	0.33σ	3.4σ
2018	1.1σ	4.2σ
2024	1.4σ	5.1σ

Input from HQET

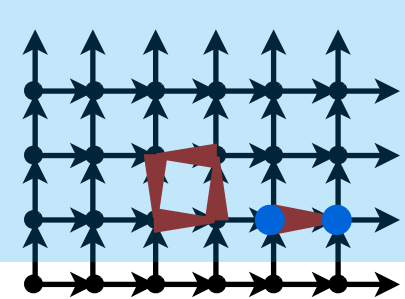
Input from LQCD

- There are several other $\sim 3\sigma$ tensions between theory & experiment in flavor physics



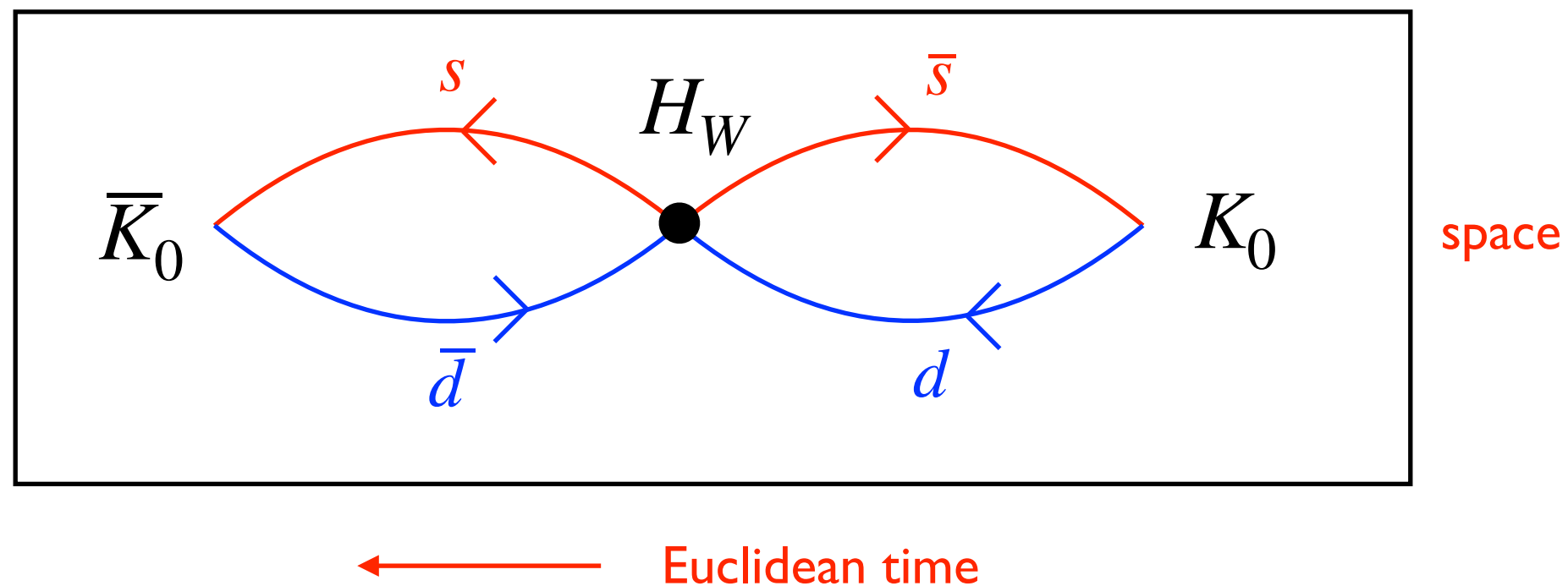
Outline

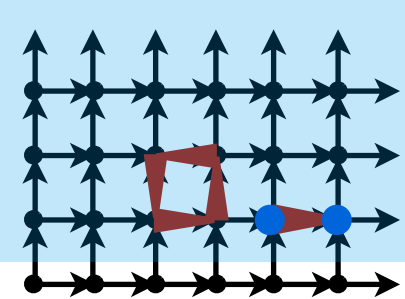
- Standard model & searching for physics beyond
- QCD & Lattice QCD
- High precision lattice QCD
- Constraining the Standard Model with LQCD
- **Extending the LQCD frontier**
- Summary



Present frontier

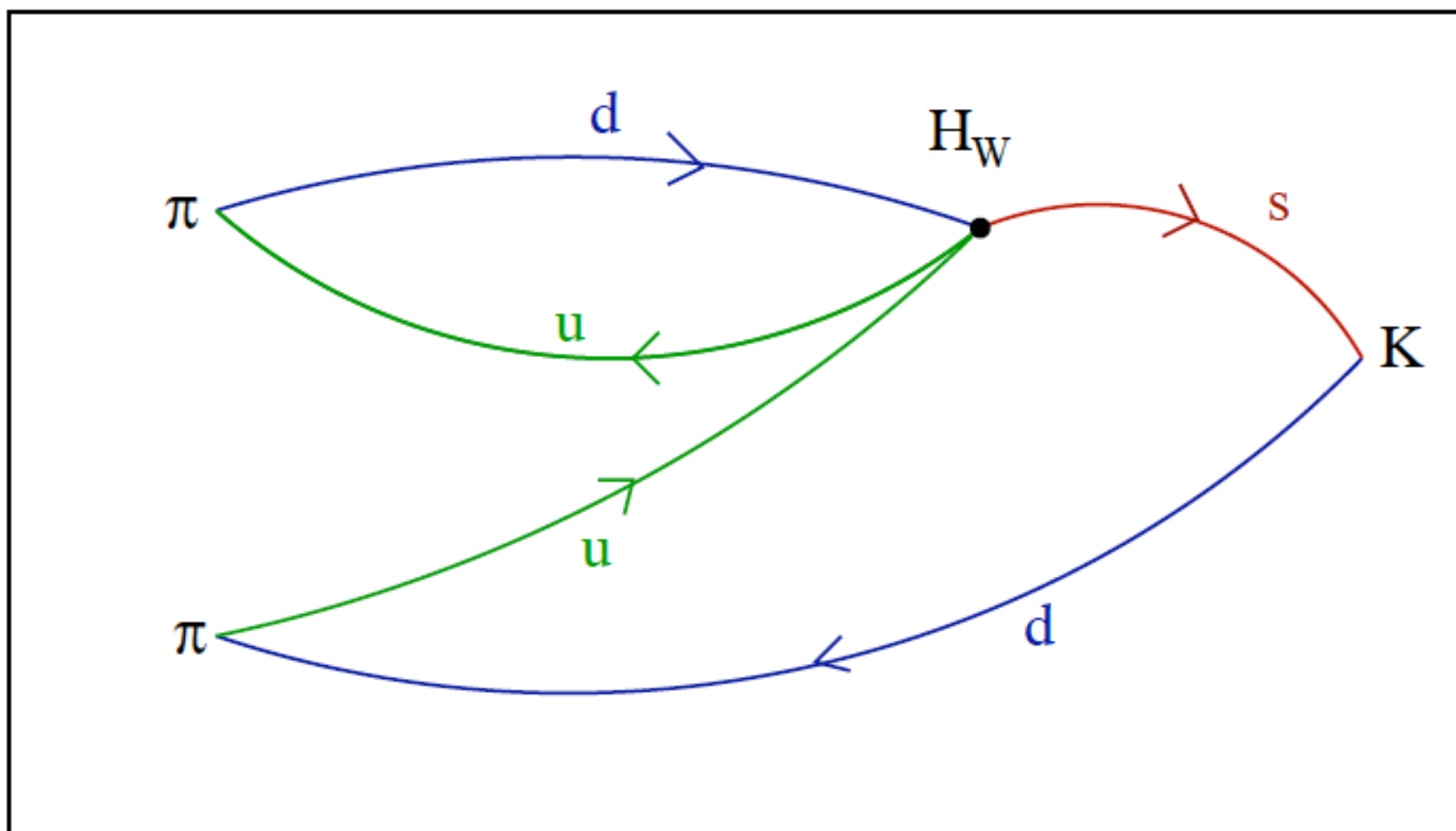
- Present “FLAG ready” LQCD calculations almost all involve single hadrons, e.g. B_K

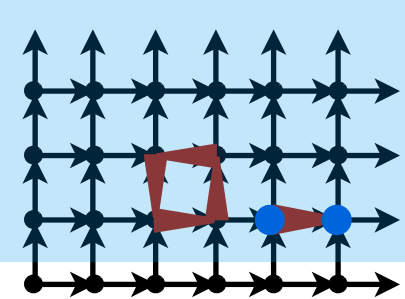




Present frontiers

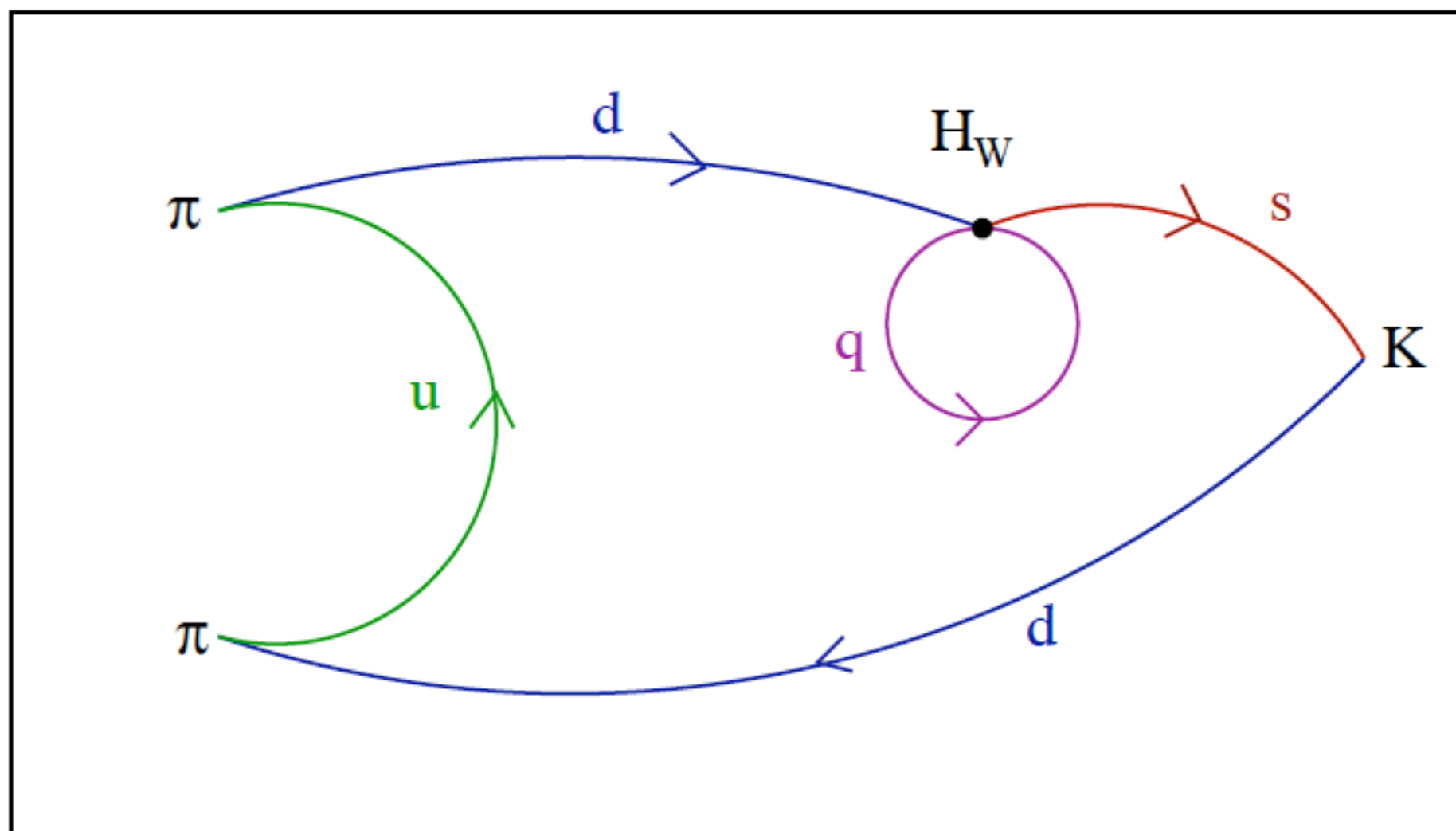
- Processes involving 2+ hadrons are more challenging
 - Major progress on $\mathcal{A}(K \rightarrow \pi\pi)$: now “FLAG-ready” with $\sim 10\%$ errors [RBC-UKQCD collaboration]

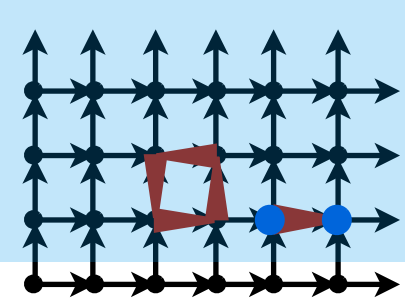




Present frontiers

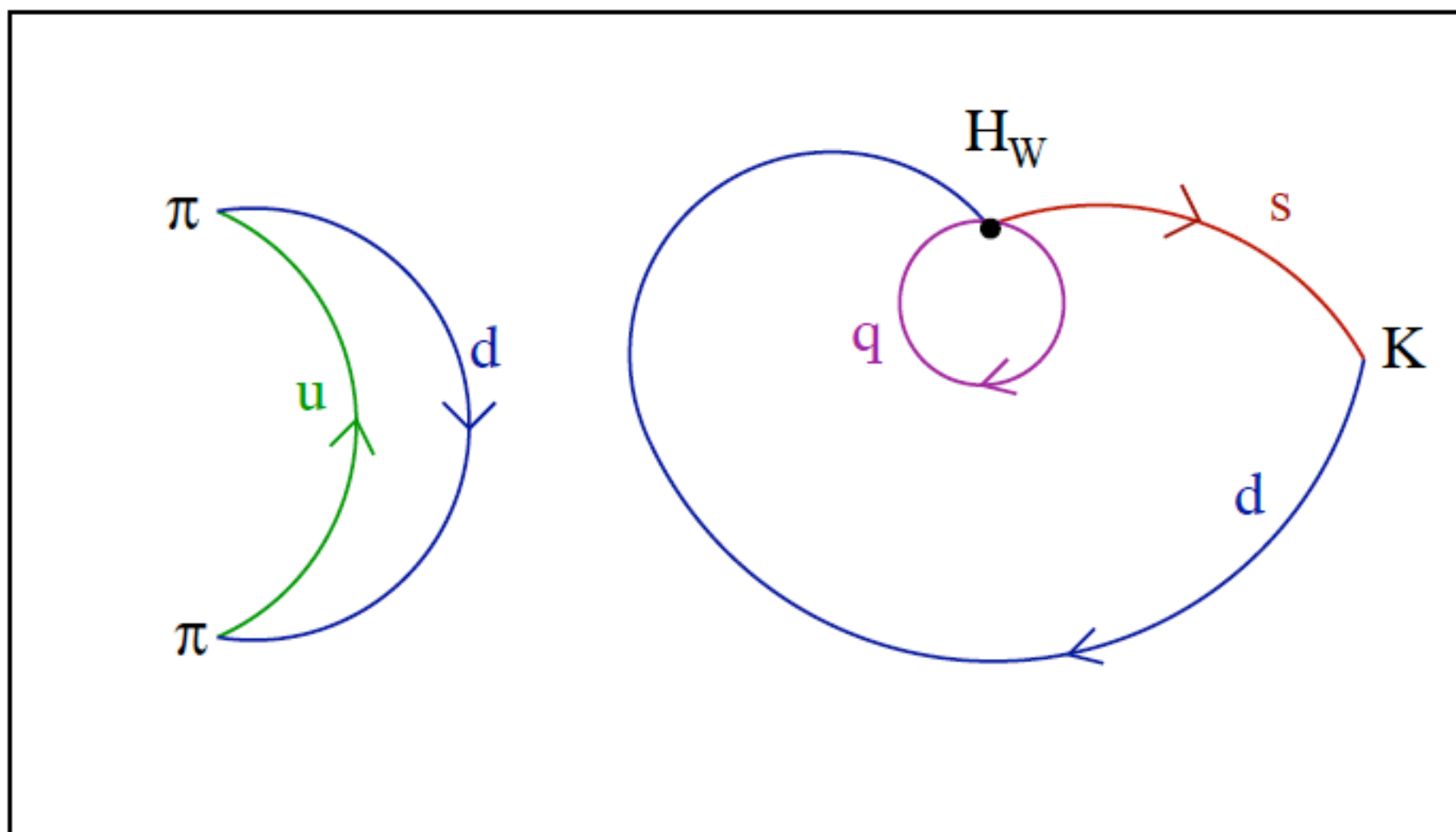
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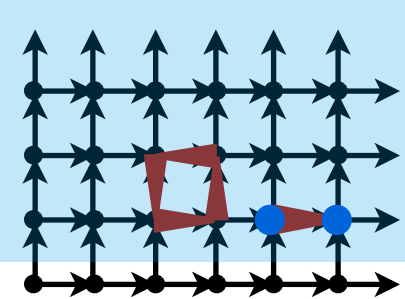




Present frontiers

- Processes involving 2+ hadrons are more challenging
 - Major progress on $\mathcal{A}(K \rightarrow \pi\pi)$: now “FLAG-ready” with $\sim 10\%$ errors [RBC-UKQCD collaboration]





Present frontiers

- Processes involving 2+ hadrons are more challenging

- Major progress on $\mathcal{A}(K \rightarrow \pi\pi)$: now “FLAG-ready” with $\sim 10\%$ errors [RBC-UKQCD collaboration]

- Quantitative understanding of $\Delta I = 1/2$ rule

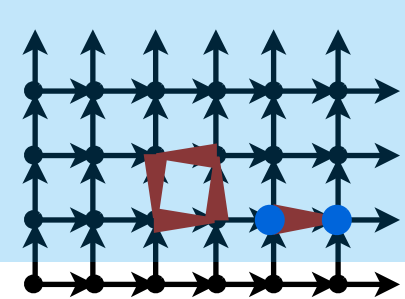
$$\frac{\Gamma(K_S^0 \rightarrow \pi\pi)}{\Gamma(K^+ \rightarrow \pi\pi)} \approx 330$$

- Quantitative prediction of CP-violating parameter ε'

- LQCD matrix elements + other inputs predict [UTfit, 2212.03894]

$$\varepsilon'/\varepsilon = 15.2(4.7) \cdot 10^{-4}$$

- Agreement with experiment [$\varepsilon'/\varepsilon = 16.6(3.3) \cdot 10^{-4}$] constrains theories of new physics



Many other frontiers

Maarten & Aida
will discuss

- Testing the Standard Model

- Isospin breaking & EM, $(g - 2)_\mu$, inclusive decay rates, Δm_K , $D - \bar{D}$ mixing, CP-violation in $D \rightarrow \pi\pi, K\bar{K}, \dots$

- Understanding the strong interactions

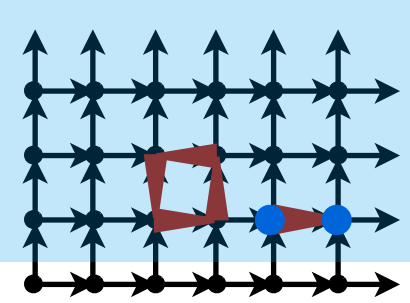
- Nucleon axial form factors, structure fcns, GPDs, TMDs, finite density, NN and NNN interactions, multiparticle scattering and exotic hadrons, ...

I will discuss

- Beyond LQCD

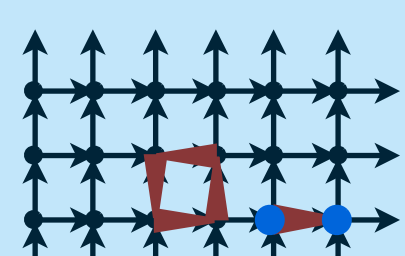
- Formulating chiral gauge theories, nearly-conformal theories with more flavors, symmetric mass generation, ...

Maarten will
discuss

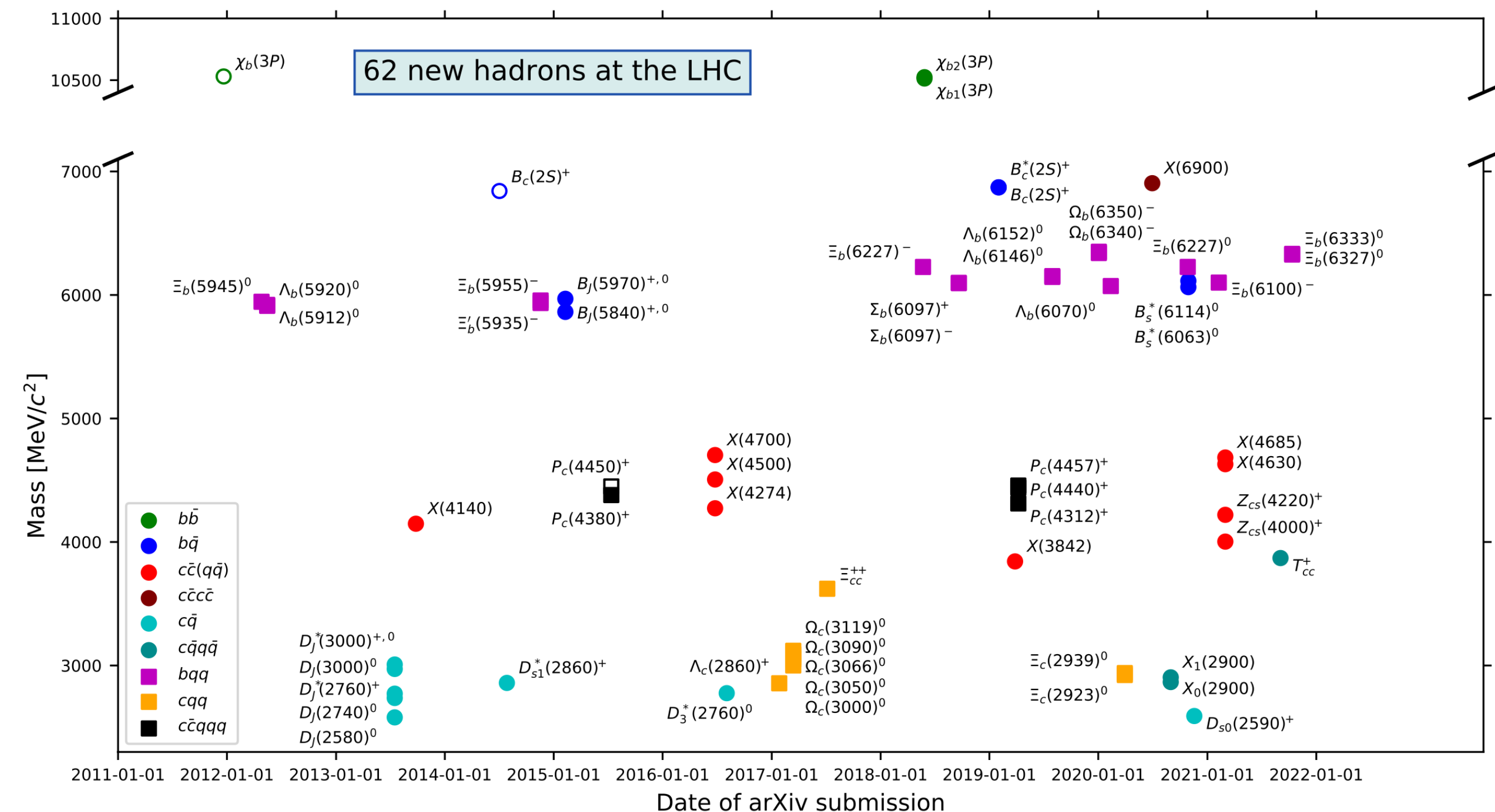


Why multiparticle amplitudes?

- Calculate properties of strong interaction resonances
 - E.g. exotics such as $T_{cc}(3875)^+ \rightarrow DD^* \rightarrow DD\pi$

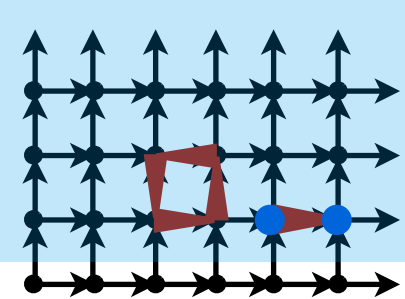


Cornucopia of resonances



[I. Danilkin, talk at INT workshop, March 23]

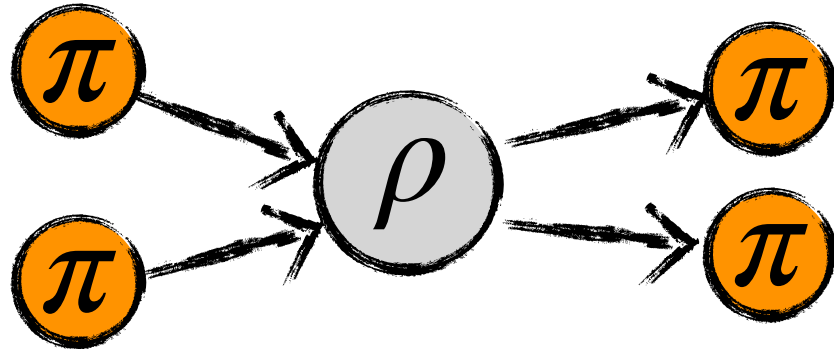
+ data from Babar, Belle, COMPASS, ...



Why multiparticle amplitudes?

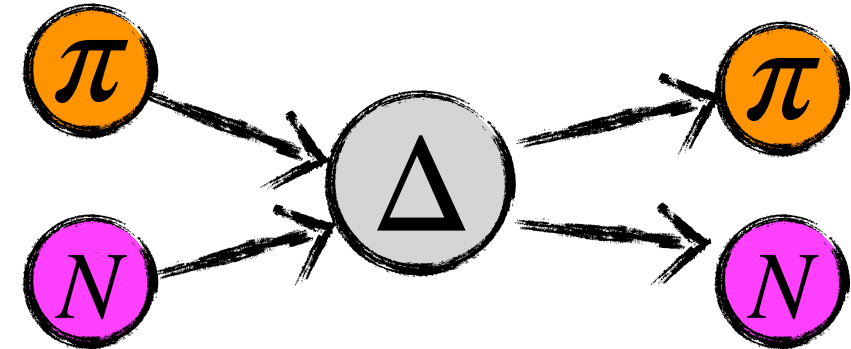
- Calculate properties of strong interaction resonances
 - E.g. exotics such as $T_{cc}(3875)^+ \rightarrow DD^* \rightarrow DD\pi$
- Determine three particle “forces” for $3n$, 3π , $3K$, ...
 - Needed for neutron star EoS, properties of large nuclei, ...
- Calculate multiparticle weak decay amplitudes
 - $K \rightarrow 3\pi$ (method known), & $D \rightarrow \pi^+\pi^-$, K^+K^- (open question)

Present status



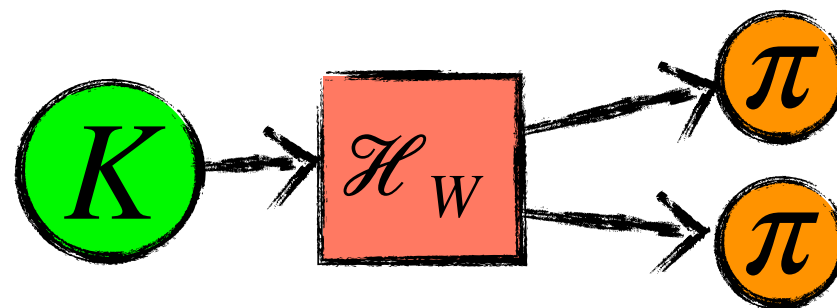
Physical quark masses

E.g.: Wang, Leinweber, Liu, Liu, Sun, Thomas, Wu, Xing, Yu,
2502.03700



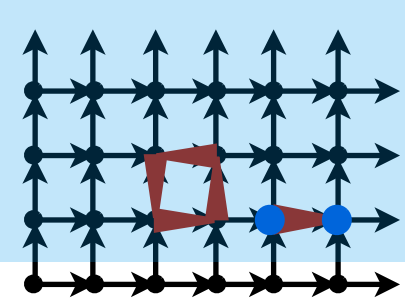
$$M_{\pi} = 200 \text{ MeV}$$

Bulava, Hanlon, Hörz, Morningstar, Nicholson, Romero-López,
Skinner, Varnas, Walker-Loud, 2208.03867



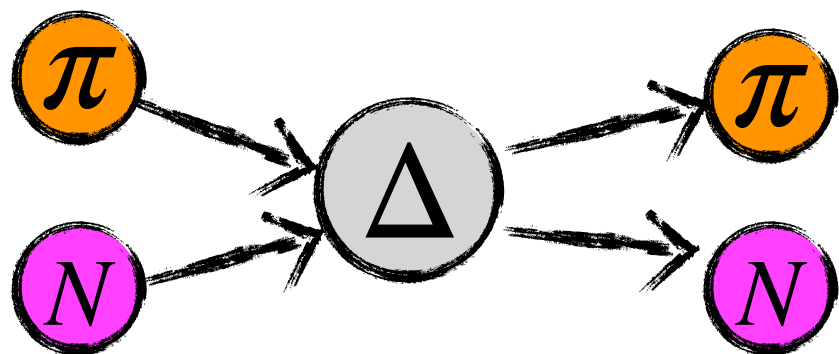
“FLAG-ready”, i.e. fully controlled

Abbott et al. [RBC-UKQCD collaboration], 2004.09440



Δ resonance

Bulava, Hanlon, Hörz, Morningstar, Nicholson, Romero-López, Skinner, Varnas, Walker-Loud, 2208.03867



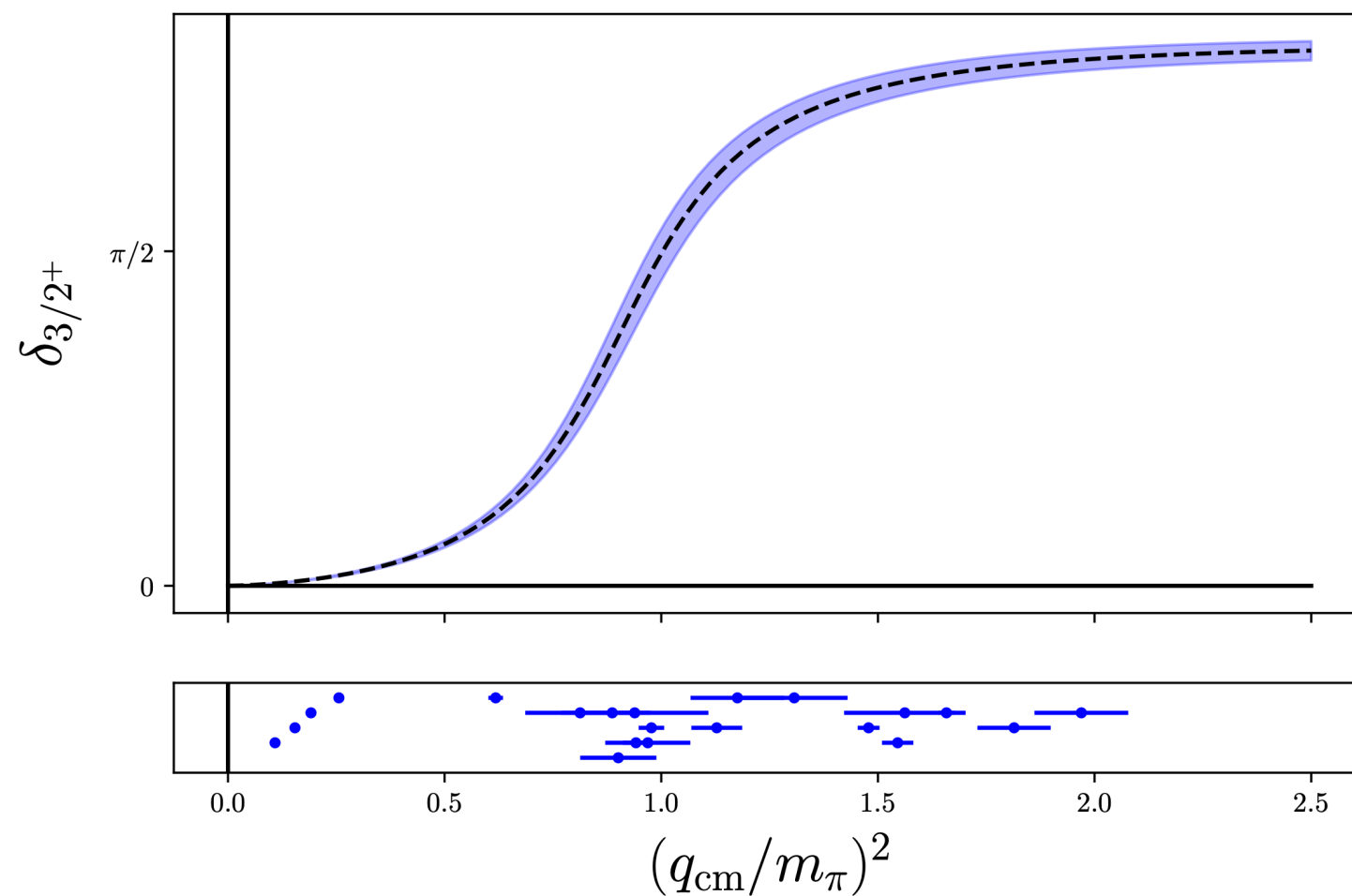
$$M_\pi \approx 200 \text{ MeV}$$

$$M_N \approx 950 \text{ MeV}$$

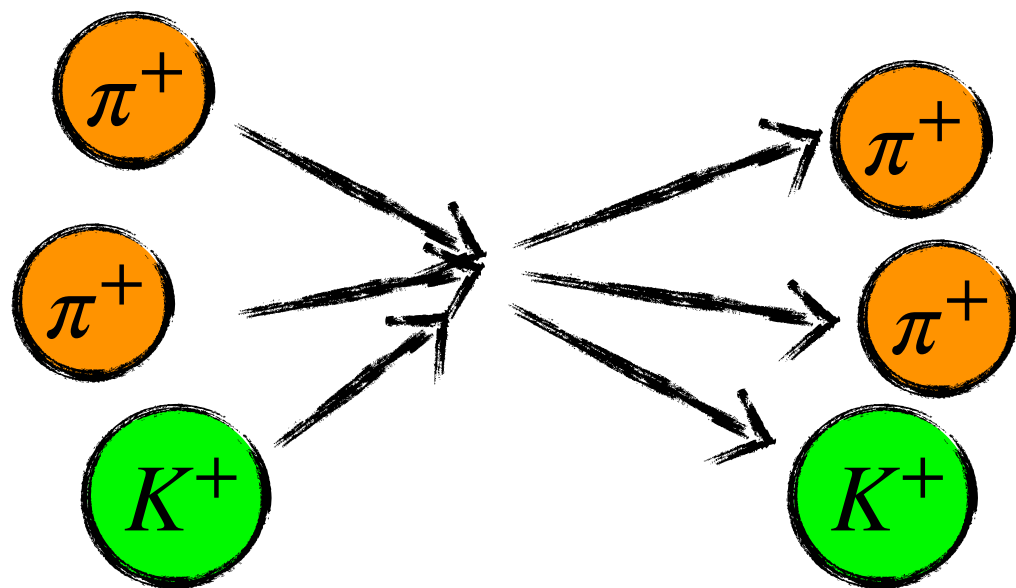
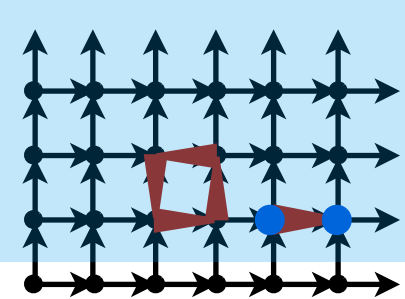
$$a = 0.063 \text{ fm}$$

$$L^3 \times T = 64^3 \times 128$$

Clover fermions (CLS)

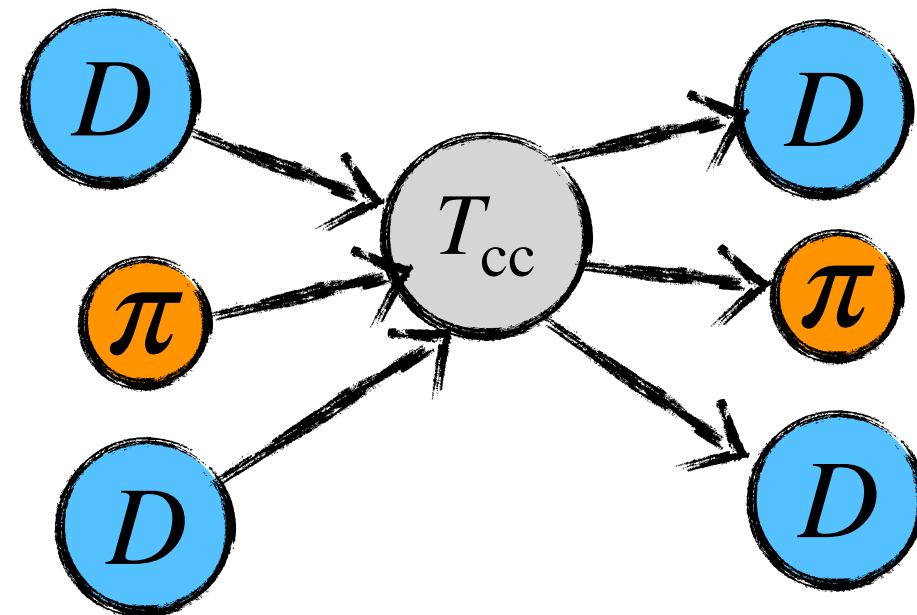


Present frontier



Physical quark masses

Dawid, Draper, Hanlon, Hörz, Morningstar, Romero-López,
SRS, Skinner, [2502.14348](#) & [2502.17976](#)

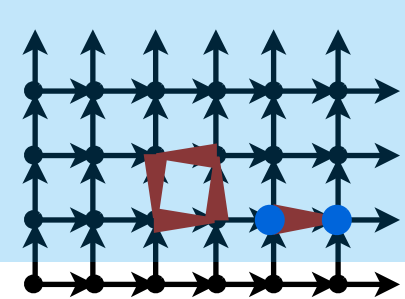


$$M_{\pi} = 200 \text{ MeV}$$

Padmanath, Prelovsek, [2202.10110](#)

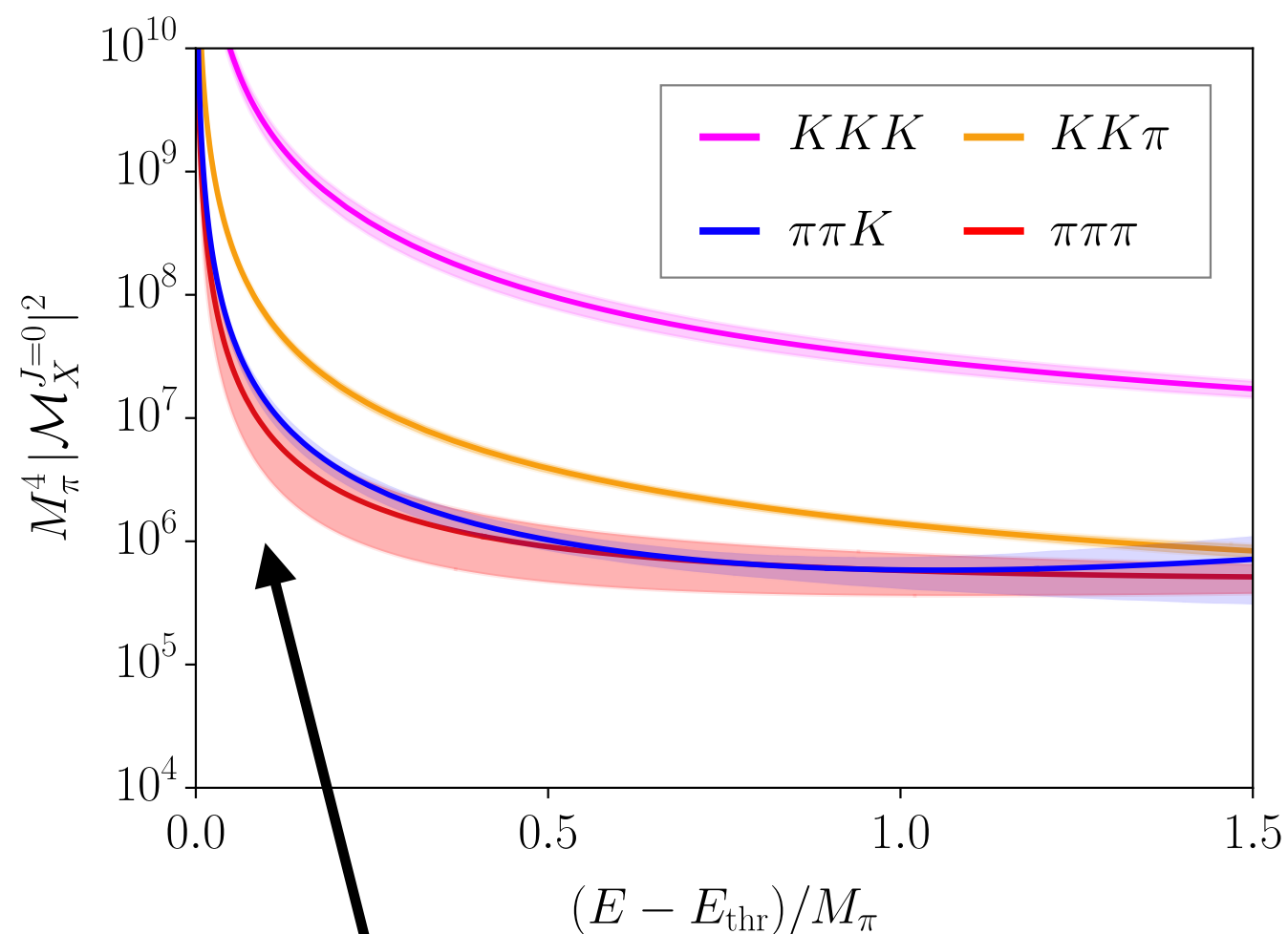
...

Dawid, Romero-López, SRS, [2409.17059](#)

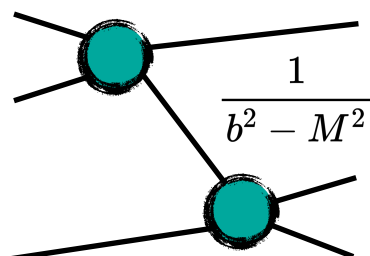


3-meson amplitudes

$$M_\pi^4 |\text{Amplitude}|^2 \quad (J^P = 0^-)$$



**Divergent
at threshold**

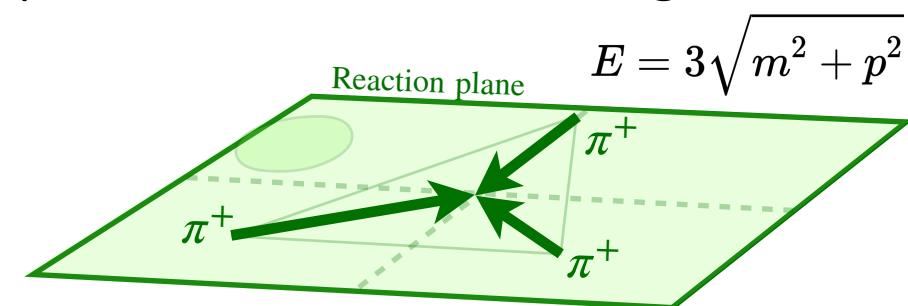


$$m_\pi = 130 \text{ MeV}, m_K = 500 \text{ MeV}$$

$$\text{Lattice spacing } a = 0.063 \text{ fm}$$

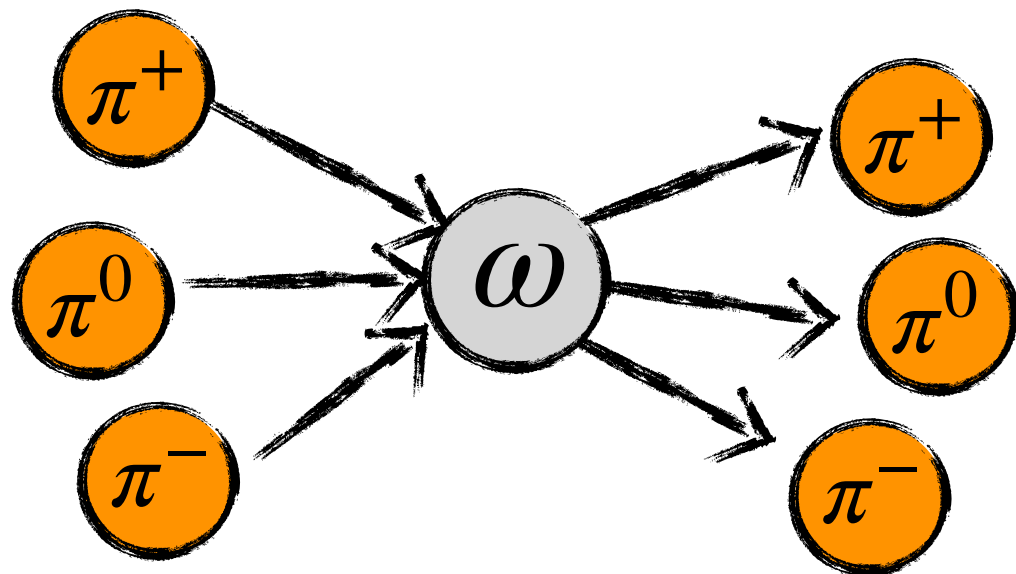
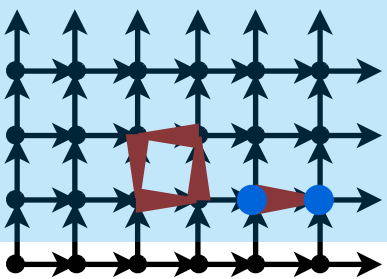
$$\text{Lattice size } 96^3 \times 192$$

Equilateral kinematic configuration

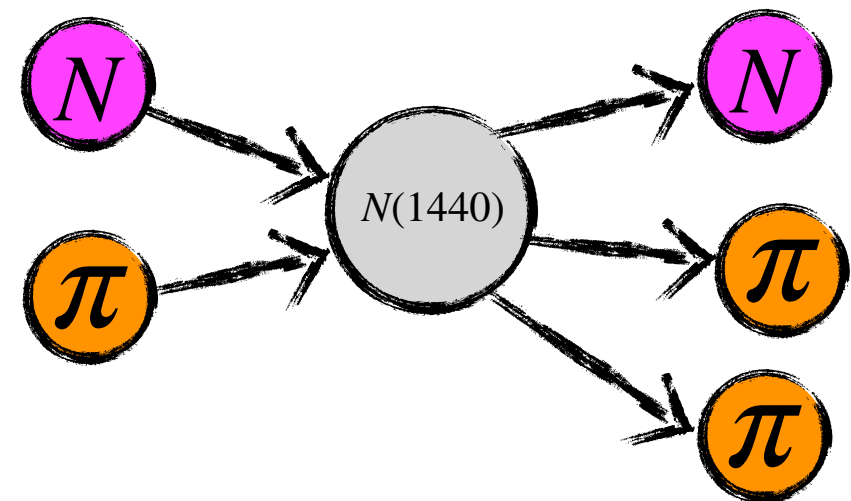


Dawid, Draper, Hanlon, Hörz, Morningstar,
Romero-López, SRS, & Skinner,
[2502.14348](#) & [2502.17976](#)

Near future

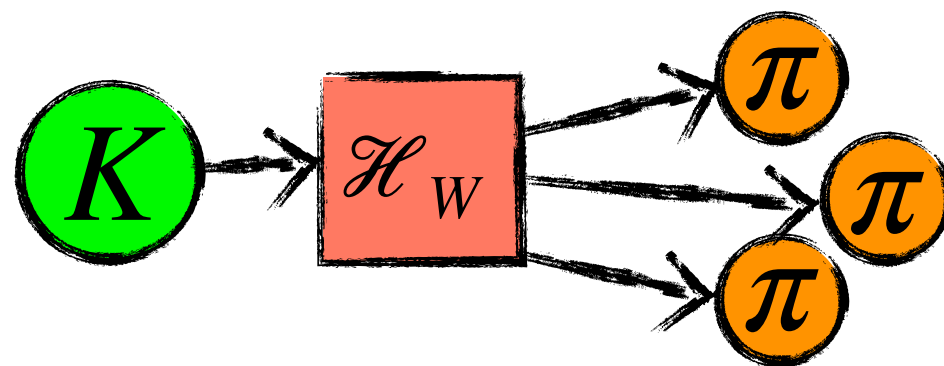


Formalism: Hansen, Romero-López, SRS, 2003.10974
 First LQCD results and alternative formalism:
 Yan, Mai, Garofalo, Meißner, Liu, Liu, Urbach, 2407.16659

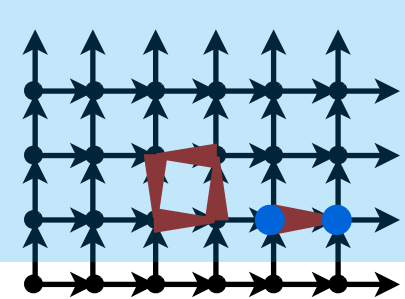


Roper resonance

Formalism: Briceño, Dawid, Hansen, Jackura, Romero-López, Smith, SRS, in prep.



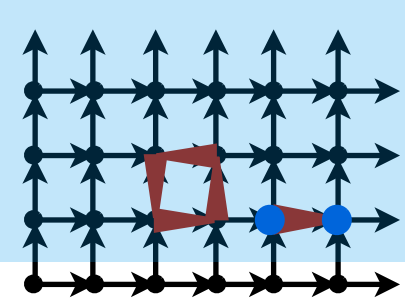
Formalism: Hansen, Romero-López, SRS, 2101.10246



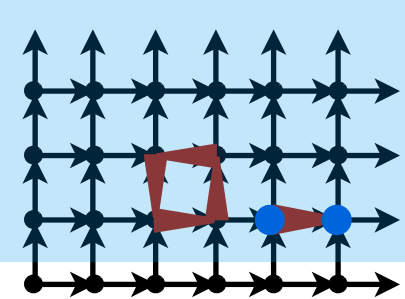
Outline

- Standard model & searching for physics beyond
- QCD & Lattice QCD
- High precision lattice QCD
- Constraining the Standard Model with LQCD
- Extending the LQCD frontier
- **Summary**

Summary

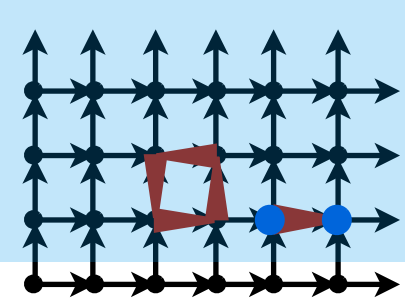


- Lattice QCD provides precise results for a large number of quantities, with complete error budgets
- It plays a crucial role in the interpretation of an increasing number of experiments looking for new physics at the “intensity frontier”



Connection to experiments

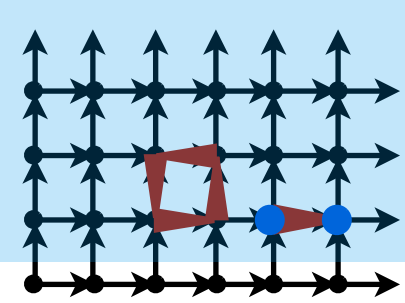
- FNAL & JPARC $(g - 2)_\mu$ —HVP & LBL
- LHCb & BELLE II—Exclusive & inclusive matrix elements
- Neutrino experiments (DUNE, ...)—axial form factors
- Rare kaon decay experiments (JPARC, CERN)—matrix elements
- Dark Matter searches (SCDMS,...)—scalar form factors,...
- Heavy Ion colliders (Alice@LHC, BNL)—phase diagram, ...
- Higgs factory—precise quark masses, α_s
- ...



Summary

- Lattice QCD provides precise results for a large number of quantities, with complete error budgets
- It plays a crucial role in the interpretation of an increasing number of experiments looking for new physics at the “frontier”
- Many new quantities that can be reliably calculated over the next 5-10 years
- If quantum computers fulfill their promise, many further extensions will be possible, e.g. to real-time quantities

The future of LQCD
is very bright!



Thank you!
Questions?